Waveform model for fully focused SAR altimetry

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Model Backscattered Power

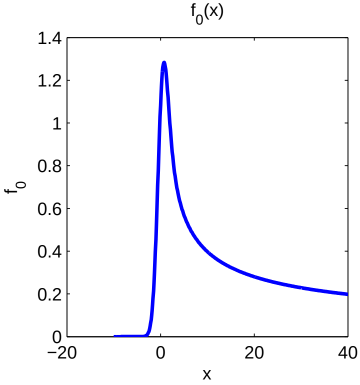
The backscattered mean power of a SAR altimeter has been modeled with some success by the following function, where G is the related to the antenna gain, H_s is the significant wave height, k is the range bin and ℓ is the doppler bin. (TGRS Feb 2015)

$$P_{k,\ell} = A\frac{G_{k,\ell}}{\sqrt{\Delta_\ell}} f_0\left(\frac{L_z k - \langle z\rangle}{\Delta_\ell}\right)$$

$$f_0(x) = \int_0^\infty e^{-(u^2-x)^2/2} du$$

$$\Delta_\ell^2 = \sigma_r^2 + (\sigma_x \theta_\ell)^2 + \left(\frac{H_s}{4}\right)^2$$

σ_r and σ_x are the range and doppler PTR standard deviation. θ_ℓ is the look angle of beam ℓ.

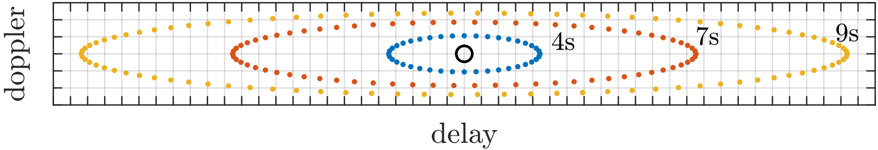


Impediment to using existing model

- Model assumes a stationary surface.

Perhaps forgivable for 4ms.
Not forgivable for 2000ms.

Here we see the progression of a point scatterer in the case of three maximum amplitude waves of the indicated period.



The position every 100ms is shown.

Generalization

We assume that scatterer's vertical motion can be sufficiently captured by a linear approximation in position and velocity.

$$z = \bar{z} + \bar{v}t$$

$$v_z = \bar{v} + \bar{a}t$$

Surprisingly the form of the model stays the same . . . but

$$\sigma_x \to (12.5\mathrm{m})\sqrt{\frac{\tau^2}{\sigma_t^2} + \frac{\bar{a}^2}{g^2}\frac{\sigma_t^2}{\tau^2}} \quad \textbf{depends on } \bar{a}$$

$$\theta_\ell \to \theta_\ell - \frac{\bar{v}}{v_t} \quad \textbf{depends on } \bar{v}$$

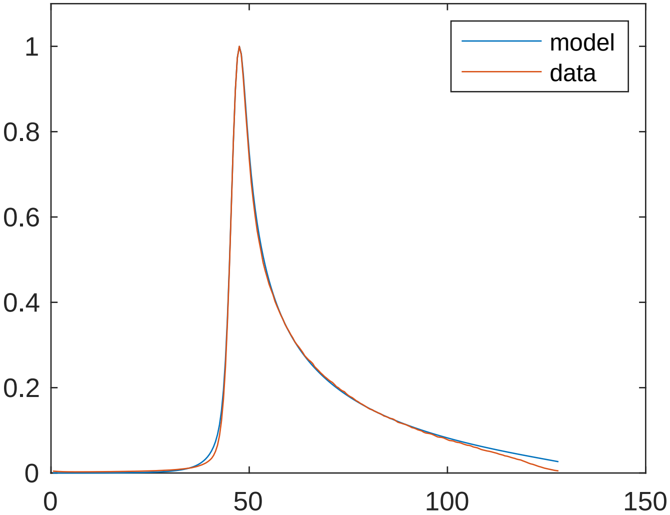
$$\langle z \rangle \to \langle \bar{z} \rangle + h\left(\theta_\ell - \frac{\bar{v}}{2v_t}\right)\frac{\bar{v}}{v_t} \quad \textbf{depends on } \bar{v}$$

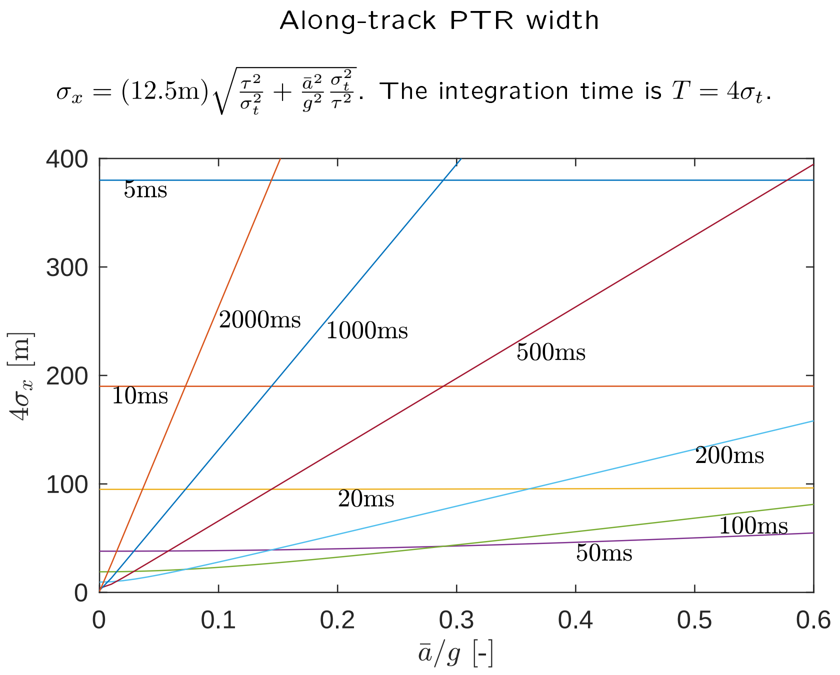
with $\tau \approx 10\mathrm{ms}$.

$$P_{k,\ell}(\bar{v},\bar{a}) = C\frac{G_{k,\ell}}{\sqrt{\Delta_\ell}} f_0\left(\frac{L_z k - \langle \bar{z} \rangle - h\left(\theta_\ell - \frac{\bar{v}}{2v_t}\right)\frac{\bar{v}}{v_t}}{\Delta_\ell}\right)$$

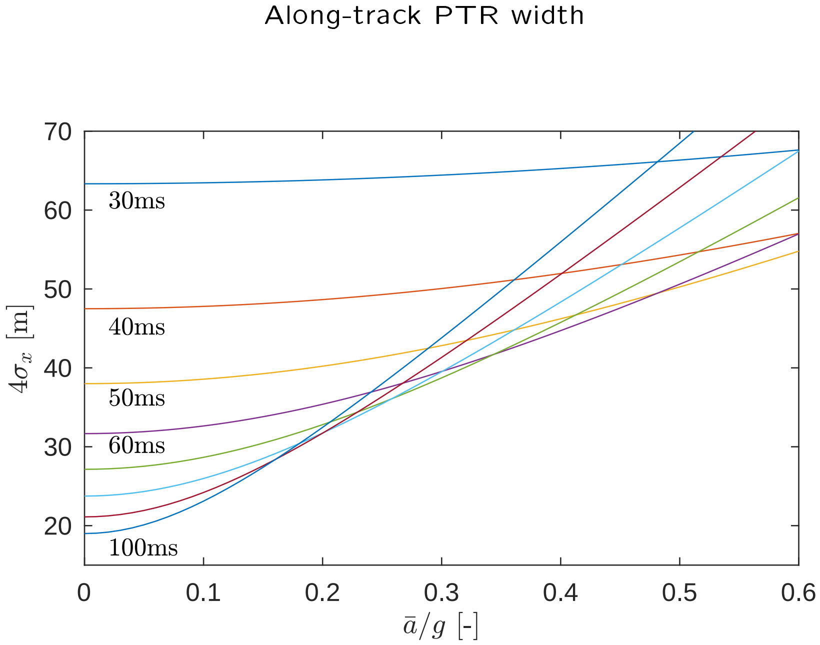
$$\Delta_\ell^2 = \sigma_r^2 + \sigma_x^2(\theta_\ell - \frac{\bar{v}}{v_t})^2 + \sigma_z^2$$

Fitting





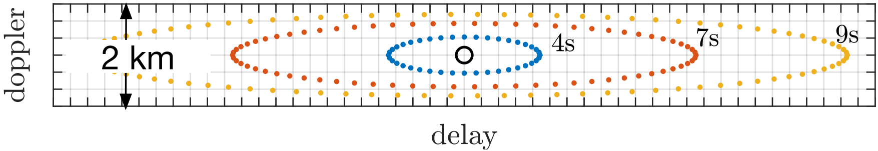
Along-track PTR width
$\sigma_x = (12.5\mathrm{m})\sqrt{\frac{\tau^2}{\sigma_t^2} + \frac{\bar{a}^2}{g^2}\frac{\sigma_t^2}{\tau^2}}$. The integration time is $T = 4\sigma_t$.
$4\sigma_x$ [m]
$\bar{a}/g$ [-]
400
300
200
100
0
0.1
0.2
0.3
0.4
0.5
0.6
5ms
10ms
20ms
50ms
100ms
200ms
500ms
1000ms
2000ms



Along-track PTR width
$4\sigma_x$ [m]
$\bar{a}/g$ [-]
30ms
40ms
50ms
60ms
100ms
70
60
50
40
30
20
0
0.1
0.2
0.3
0.4
0.5
0.6

What does a resolution of 50 meters mean?

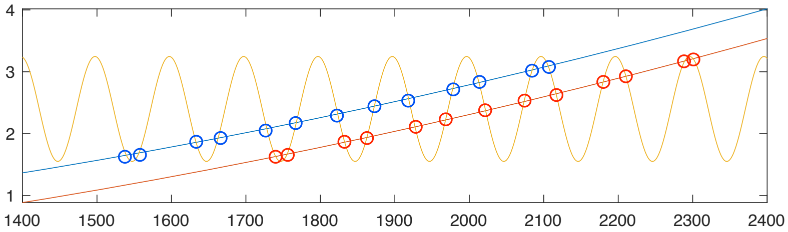
Recall the history of a single scatterer.



How can we say we have a doppler resolution of 50 meters (let alone 300 meters) when the scatterers we have “focused” are spread over 2 km?

The answer to this is something that we all know, but upon which we might not often reflect.

Accustomed to this in range.



• We resolve in range and then imagine that we have resolved in across track distance.

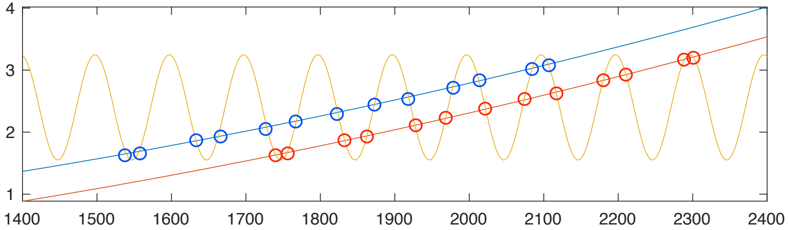
• But there is much overlap in across track position between two adjacent delay bins.

Even so these adjacent bins are not as correlated as one would expect for the almost complete overlap of their across track position.

Conjecture

What matters is the degree to which a particular bin of our “image” contains unique scatterers.

It is irrelevant to speckle noise reduction if the scatterers in a bin are not located in the same geographical location.



Conjecture

The same is true with doppler resolution.

For reduction in speckle noise what is important is the degree to which we can separate scatterers by their radial velocity.

This is what is measured by σ_x. Indeed it is better to think in terms of the velocity resolution.

$$\sigma_v = \frac{v_t}{h}\sigma_x \approx \frac{\sigma_x}{100 \text{ sec}}$$

What does a resolution of 50 meters mean?

$$\sigma_v = \frac{v_t}{h}\sigma_x$$

A 50 meter doppler resolution means a resolution of 50 cm/s in velocity.

The radar will be able to put scatterers with velocities differing by 50 cm/s in different bins.

Conclusions

• The doppler resolution is on the order of tens of meters (10 cm/s in radial velocity) depending on the surface acceleration.

• The waveform model that was developed for 3.5ms bursts has been adapted for longer bursts.

A friendly reminder: For the sake of speckle reduction, “focusing” is a partitioning of scatterers, and “resolution” is the size of the partitions.

Aknowledgments

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Thank you for your attention