Simultaneous Estimation of Tides and Topography in the Weddell Sea

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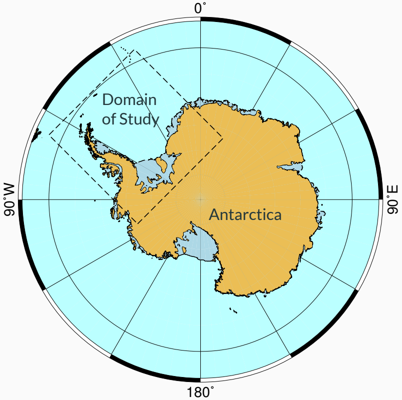
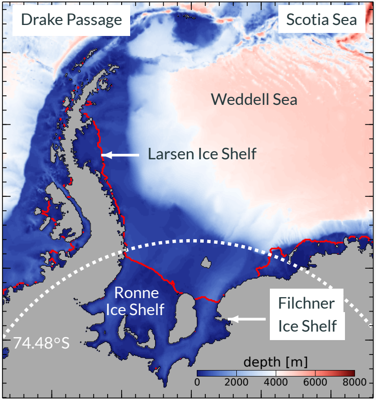
Introduction





Motivations for study: (1) tidal dissipation coupled with water mass formation & ice shelf thermodynamics, (2) tide models disagree at the level of 3 to 10cm, and (3) how to utilize latest CryoSat-2 data poleward of 66°S.

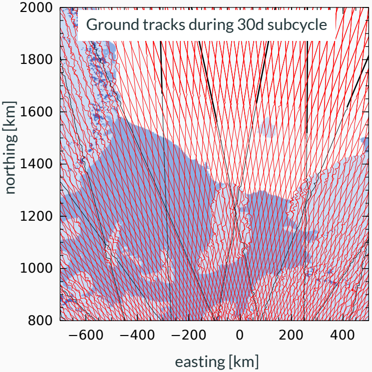
Outline

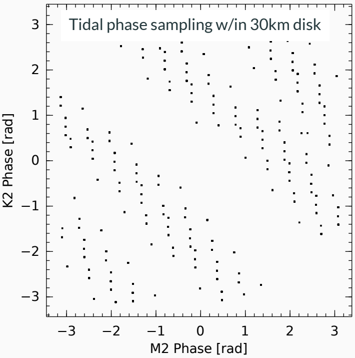
Goals:

Use variational assimilation to estimate tides (M_2, S_2, K_1, O_1) and water column thickness from CryoSat-2 data.

1. CryoSat-2 data.
2. Dynamical modeling of barotropic tides: dissipation.
3. Estimating bottom topography.

CryoSat-2 data: long-repeat orbit (368d)





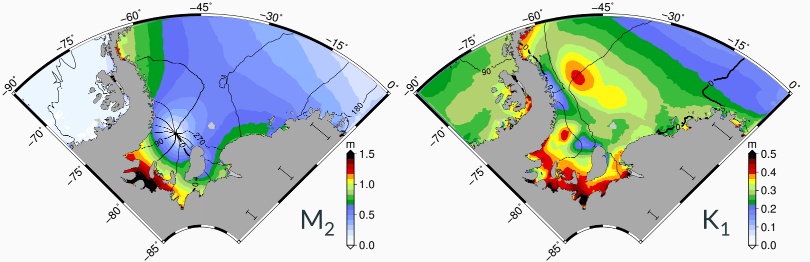
Using ESA L2 baseline-C release.

Spatial binning over 100km to 300km scales, depending on latitude, is necessary to estimate tides with centimeter precision for M_2 and K_1.

For details on tidal analysis of CryoSat-2 data, see Zaron, JPO, 48:975–993, 2018.

General features of Weddell Sea tides

Estimated from combined spatial regression and harmonic analysis of CryoSat-2 (Zaron, 2018):



Approximately 3 cm error relative to a small number (N=20) of *in situ* tidal measurements.

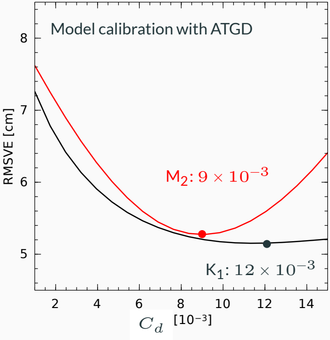
K_1 is subinertial and intensified along the continental slope; otherwise, tides are larger near the back of the ice shelves.

Dynamical modeling of barotropic tides

$$\frac{\partial \eta}{\partial t} + \nabla \cdot \mathbf{U} = 0 \tag{1}$$

$$\frac{\partial \mathbf{U}}{\partial t} + f \times \mathbf{U} + gH\nabla(\eta - \Phi) + \underbrace{\boxed{C_d u_f \frac{\mathbf{U}}{H}}}_{\text{interfacial drag}} + \underbrace{\boxed{\mathbf{F}}}_{\text{new terms}} = 0 \tag{2}$$

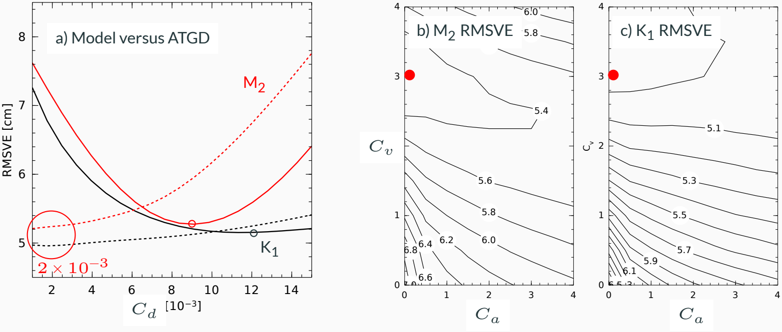
Solutions are sensitive to drag coefficient, C_d (doubled under ice shelves), and water column thickness, H.



Calibrate drag and dissipation coefficients

New linear drag terms:

$$\mathbf{F} = C_a \underbrace{\frac{|\mathbf{u} \cdot \nabla H|}{H}}_{\text{tidal displacement}} \mathbf{U} + C_v \underbrace{|\nabla \times \mathbf{u}|}_{\text{tidal vorticity}} \mathbf{U} \qquad (3)$$



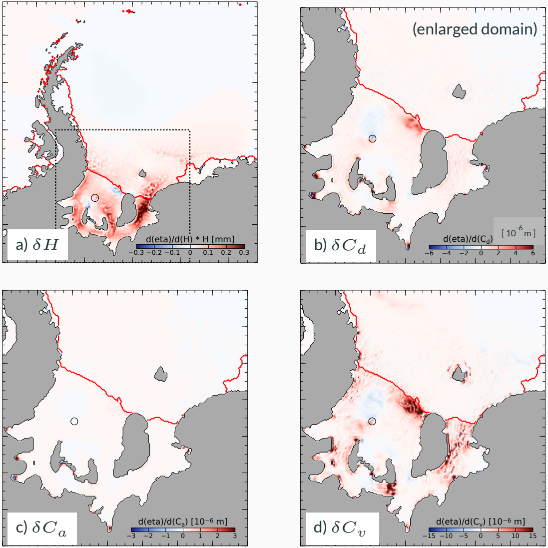
Lower error and more plausible C_d obtained using $C_a = 0$ and $C_v = 3$.

Adjoint sensitivity to H, C_d, C_a, C_v

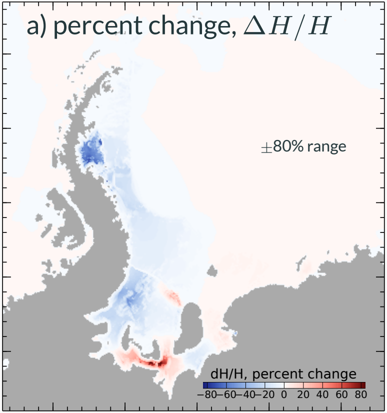
Sensitivity, e.g., $\partial\eta/\partial H$, expressed as m per unit change.

Sensitivity to H is 100 to 1000 times larger than to C_d, C_a, or C_v.

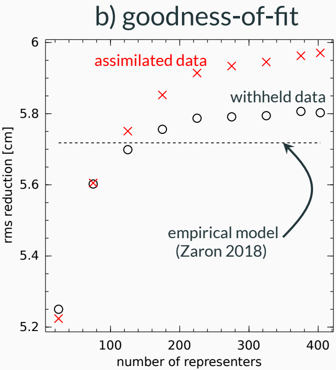
Proceed using reduced basis approach (Egbert and Erofeeva) and assimilate CryoSat into the tide model using H as control variable.

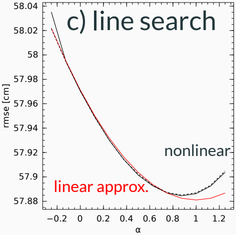


Correcting the Bottom Topography



3.5 cm rms reductution in misfit w.r.t. CryoSat-2 data.



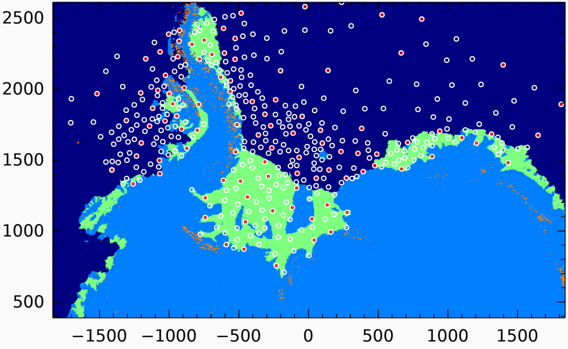


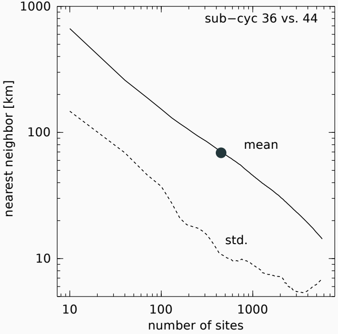
Conclusions & Summary

1. CryoSat-2 data is providing new information about tides in the Weddell Sea.
2. Variational assimilation can provide corrections to under-ice-shelf topography/water column thickness.
3. New parameterization of dissipation deserves more study.
4. Next steps: corroborating evidence (ΔH), diagnose energetics

Extra #0: "Reduced Basis" from CryoSat-2 Crossovers

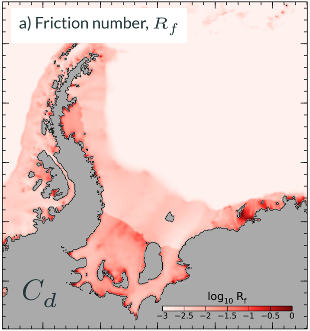
Crossover sites are subsampled and sorted by nearest neighbor distance (scaled by gH) to build a nested partition of representer sites.

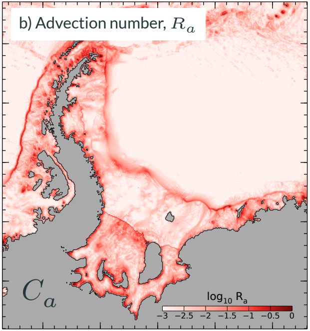




500 sites used — approx. 80km mean separation.

[For details, see script at baburu:WeddellSea/Julia/c2xo.jl.]

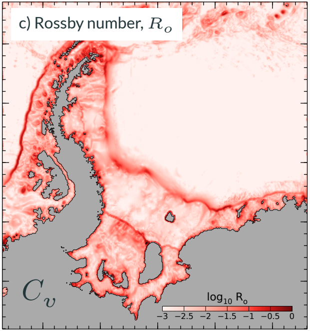




Extra #1:

Sizes of drag terms.

log-scaled, 3 orders of magnitude color scale



Extra #2: The Adjoint System

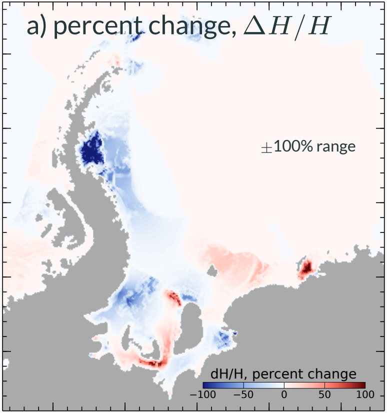
$$i\omega\zeta - \nabla \cdot \left(g\overline{H}\mu\right) = -\kappa_i$$

$$i\omega\mu - f \times \mu - \nabla\zeta + \overline{C}_d u_f \frac{\mu}{\overline{H}} + \overline{C}_a \frac{||\mathbf{v}, \nabla\overline{H}||}{\overline{H}}\mu + \overline{C}_v \varpi\mu = 0$$

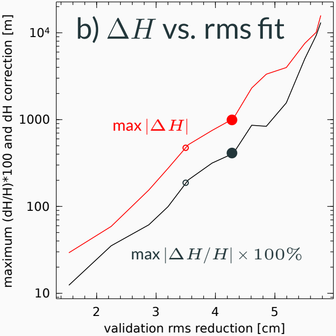
$$\lambda = -g\mu^* \cdot \nabla(\overline{\eta} - \Phi) - c_d \frac{\overline{C}_d}{\overline{H}} - c_a \frac{\overline{C}_a}{\overline{H}} + \nabla \cdot \left(\overline{C}_a \frac{[\mathbf{v}, \nabla\overline{H}]}{\overline{H}}\mu^* \cdot \overline{\mathbf{U}}\right)$$

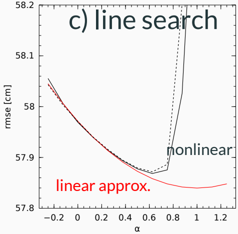
$$c_d = -u_f \frac{\mu^* \cdot \overline{\mathbf{U}}}{\overline{H}} \qquad c_a = -\frac{||\mathbf{v}, \nabla\overline{H}||}{\overline{H}}\mu^* \cdot \overline{\mathbf{U}} \qquad c_v = -\varpi\mu^* \cdot \overline{\mathbf{U}}$$

Extra #3: More Aggressive Minimization



4.0 cm rms reductution in misfit w.r.t. CryoSat-2 data.





Extra #4: Other details

1. Needed to exclude a lot of CryoSat-2 data near boundaries during the 20Hz-to-1Hz data reduction — LS estimator does not tolerate outliers/wide-tailed data distribution. 2 million data points used.
2. Solver is nonlinear — iterate on u_f and $\varpi = |\nabla \times \mathbf{u}|$.
3. Some terms neglected in the TLM & ADJ models: u_f and ϖ dependence on $(\mathbf{U}, H)$, consistent with above iteration.
4. Spatial covariance model for H: 20% error under ice shelves, 1% error offshore, correlation scale equal to minimum of 200 km or $H/|\nabla H|$. Correlation scale also shrinks to zero at material boundaries and the ice shelf edge.
5. Identification of smaller scale topography is computationally feasible but more nonlinear and ill-conditioned.