An attempt to map mesoscales and internal tides in a single massive inversion

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Context

- In an effort to improve multi-altimeter maps (DUACS system) we developed a prototype of variationnal mapping to better handle a wide range of scales in time and space

- The signatures of Internal Tides (IT,) contaminating the Mesoscale (MS) estimates, can be estimated in the same inversion by considering time-coherent modes on a multi-year window

- With the improved mapping procedure , an IT field (discarded from the maps) can be estimated

- We propose here to examine the IT field and ask whether it could be of interest to the tidal community ?

Switching to variationnal approach

- Standard OI formula to map altimetry : $\mathbf{x_a} = \mathbf{BH^T(HBH^T + R)^{-1}y}$

Estimate — (grid,obs) signal cov. — (obs,obs) signal cov. — (obs,obs) error cov — SLA obs

Main issue: prohibitive cost (α n^3) if we extend time window to include a wide range of signals. Limited to typically 1,000km, 30 days in DUACS

- We propose a variational approach involving the minimization:

$$J = \boldsymbol{\eta}^{\mathrm{T}}\mathbf{Q}^{-1}\boldsymbol{\eta} + (\mathbf{y} - \mathbf{G}\boldsymbol{\eta})\mathbf{R}^{-1}(\mathbf{y} - \mathbf{G}\boldsymbol{\eta})^{T}$$

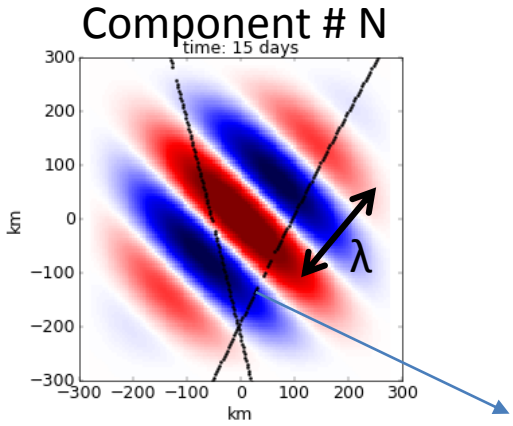
State in param space — Distinct MS and IT basis of components — Prescribed variance of the components

Benefits: We can extend the inversion window (cost α n) up to decades and introduce IT coherent components

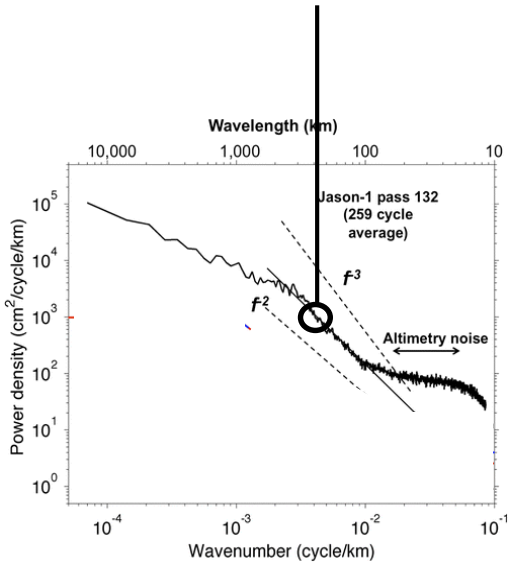
- ✓ Equivalence with OI (provided GQG^T matches the same covariance model)
- ✓ With this setup, MS and IT can be optimally estimated with respect to their covariances
- ✓ The solution strongly relies on a suited basis of components for MS and IT (see next slides)

A 3D wavelet basis for mesoscales

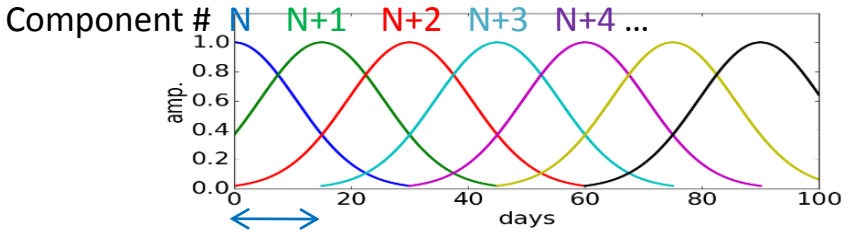
In space:



Amplitude-match with the observed mesoscale altimetry spectrum specified in **Q**

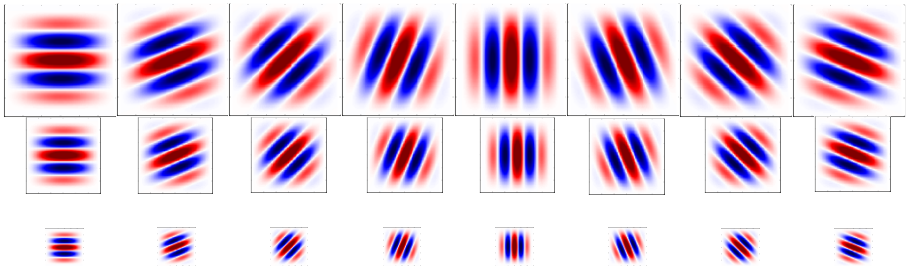


In time:



A column of **G** matrix is the value of the component at obs point.

- A full decomposition over a 30°x30° domain , 20 years, for λ between 1,000km and 80km gives **O(10^8) individual comp.**



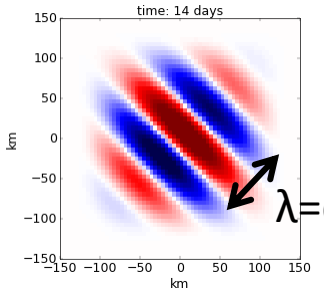
- We don't want to write **G** !! (nobs x ncomp) but only its non-zero segments sequentially for each column to get the product $\mathbf{GR}^{-1}(\mathbf{y}-\mathbf{G\eta})$

A plane wave basis for M2 Internal Tides

A local plane-wave basis, following Zhao et al., 2016, is considered for mode 1 and mode 2

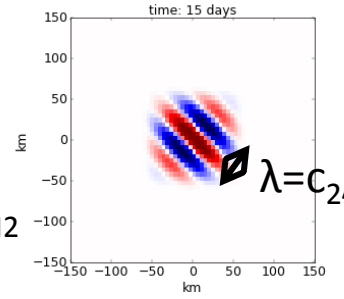
In space:

Mode-1



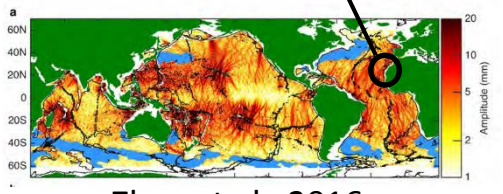
$\lambda = c_1 / f_{M2}$

Mode-2



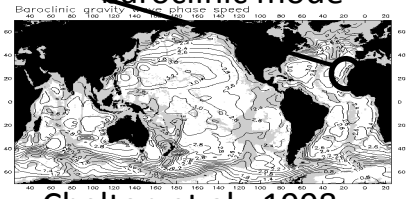
$\lambda = c_2 / f_{M2}$

Expected IT variances specified in **Q**



Zhao et al., 2016

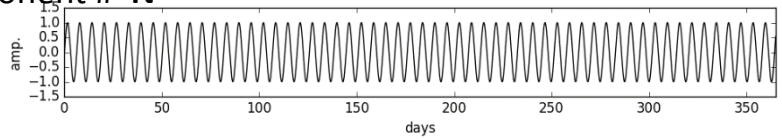
Phase-speed of 1&2 baroclinic mode



Chelton et al., 1998

In time:

Component # **N**



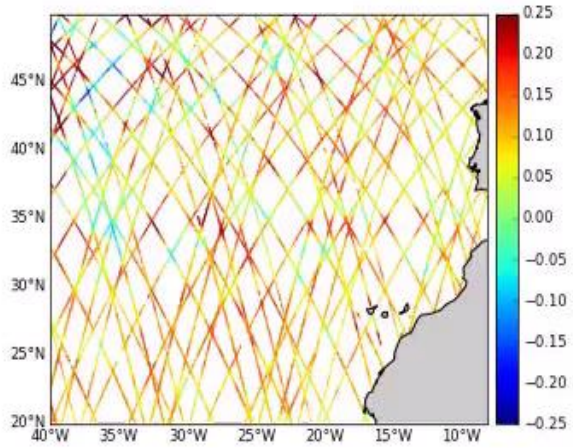
- A decomposition over a 30°x30° domain, for mode-1 and mode-2, gives **O(10^5) individual comp.**

- The non-zero segments of the **G** columns are much longer (pluri-annual comp.) but the number of modes is low: we can still easily compute $\mathbf{GR}^{-1}(\mathbf{y} - \mathbf{G\eta})$ sequentially

Running the MIOST processing system

(Multi-scale Inversion of Ocean Surface Topography)

Multi-altimeter along-track SLA



Variationnal

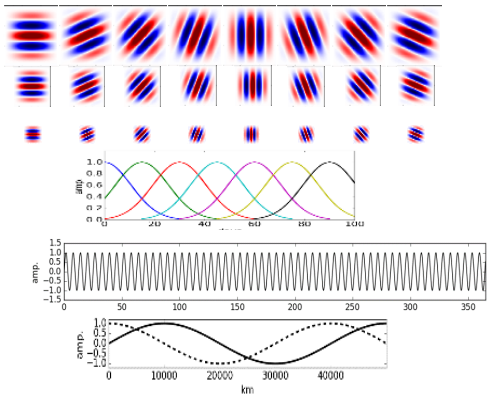
0(100) iterations

Minimization

Solution **η**:

5.10^8 adjusted parameters

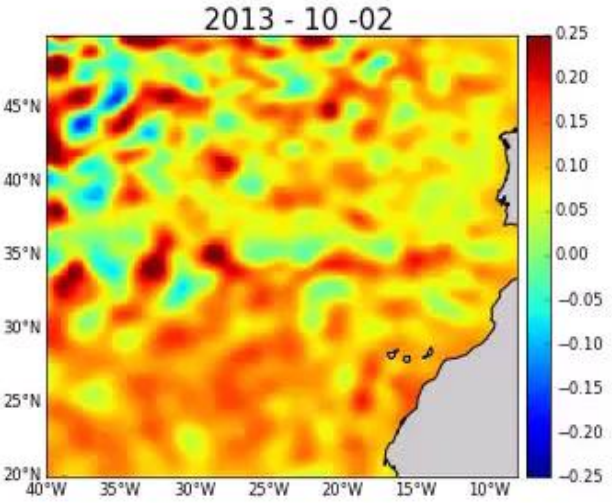
Component basis



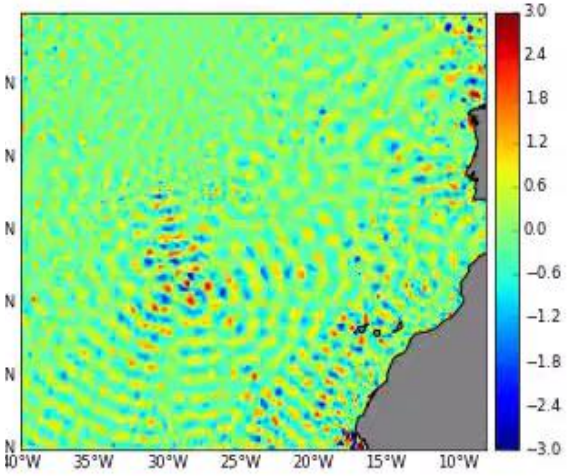
Projection in physical space

20-year worth of :

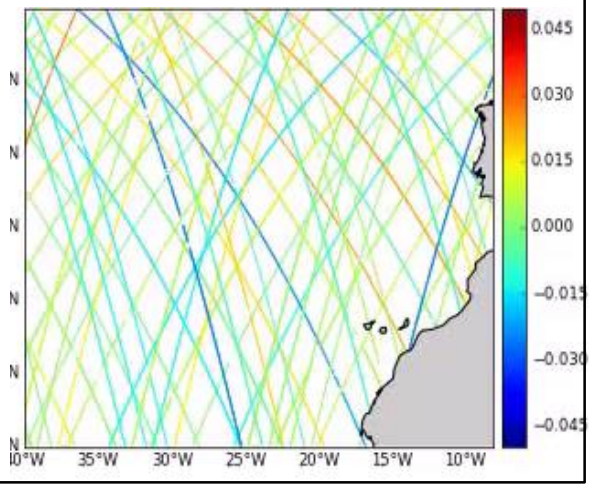
mesoscale maps



Internal-tides



Other modes



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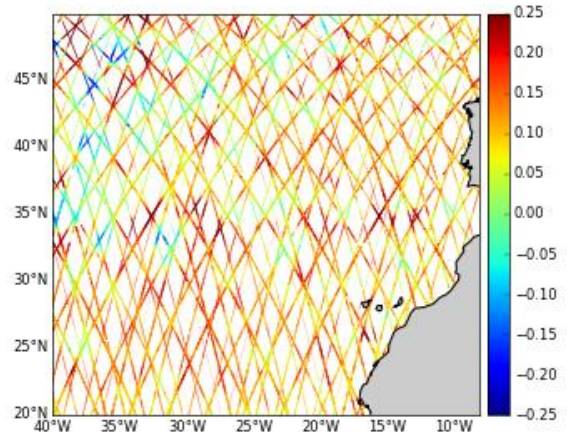

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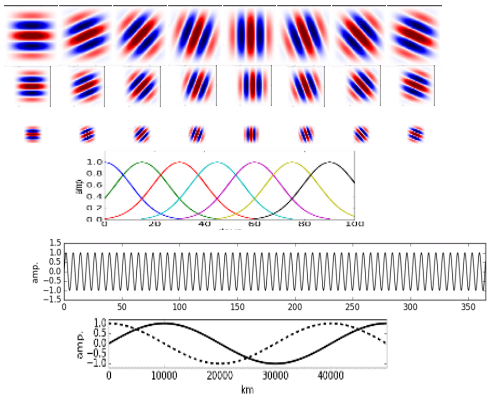

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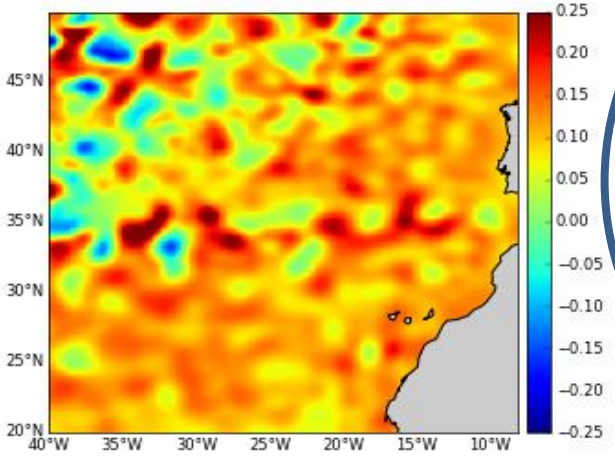
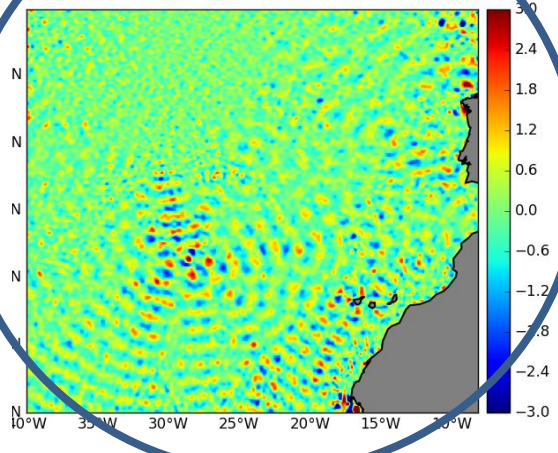
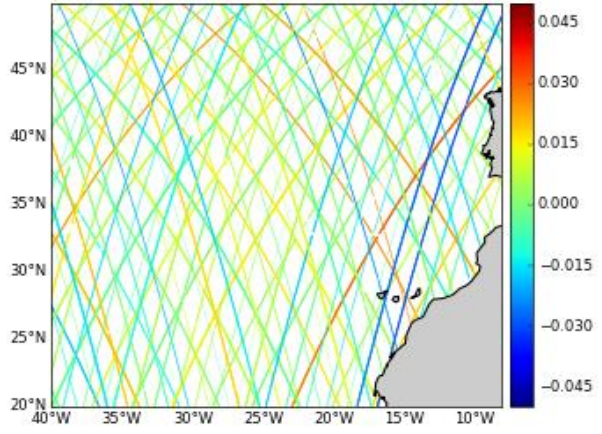


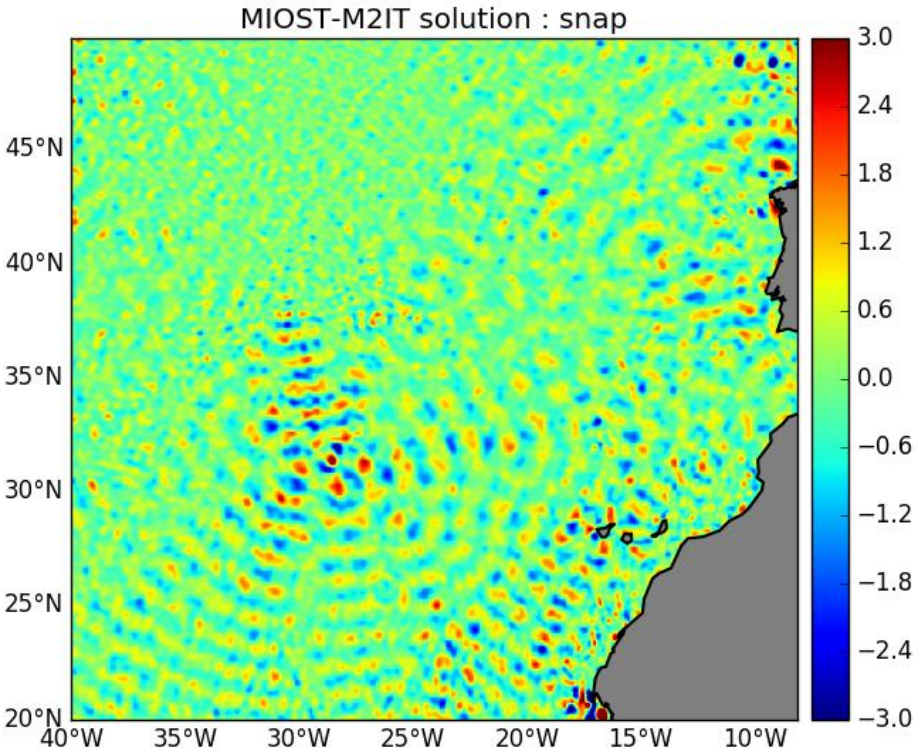
Internal-tides

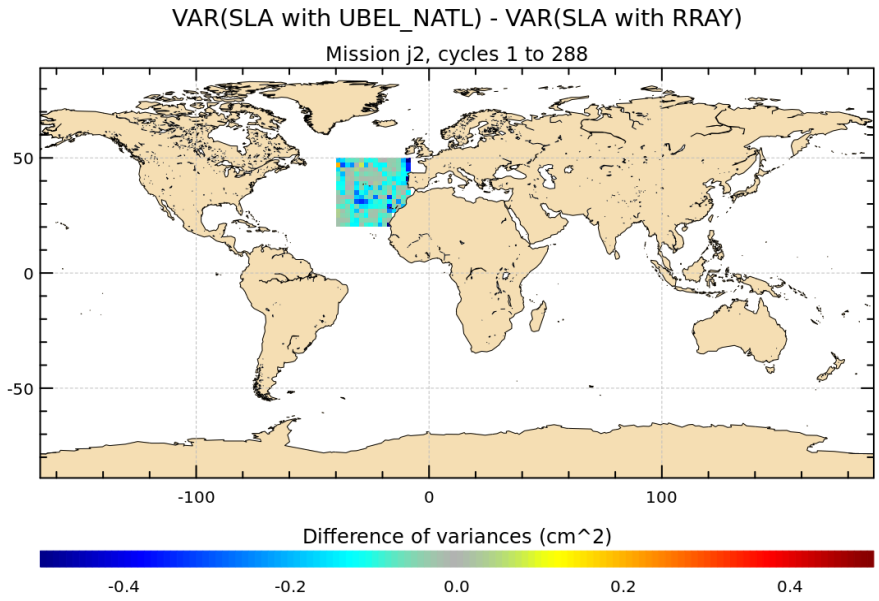


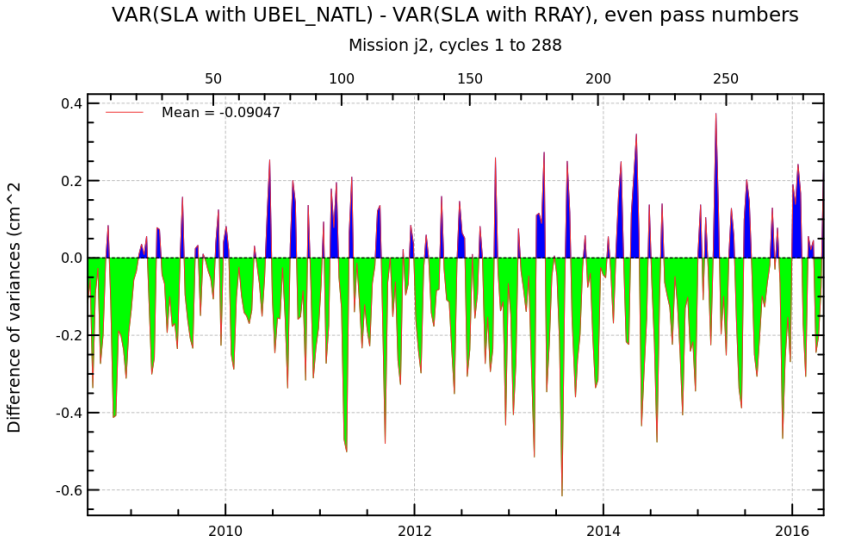
Other modes



Assessment of the IT field

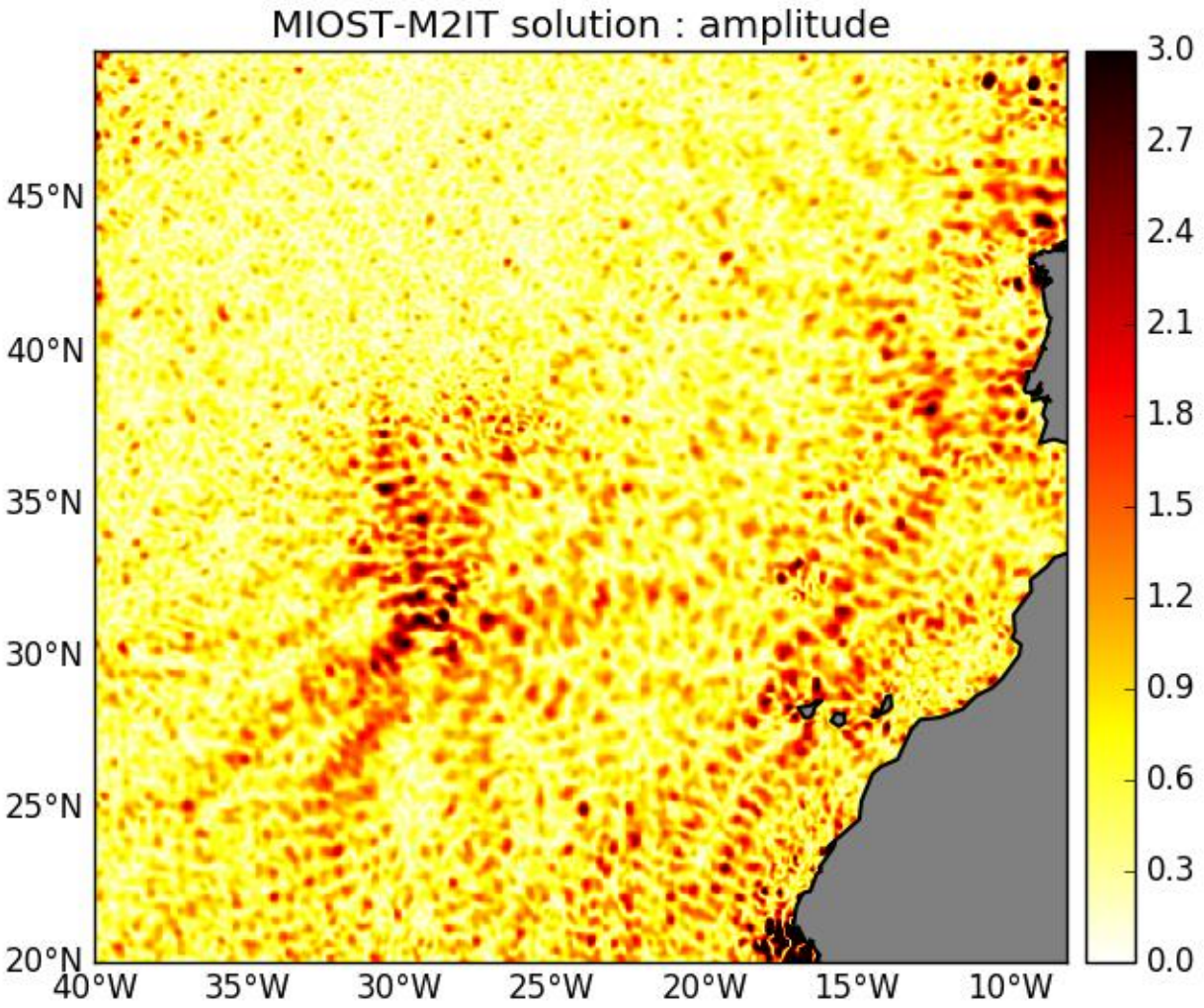




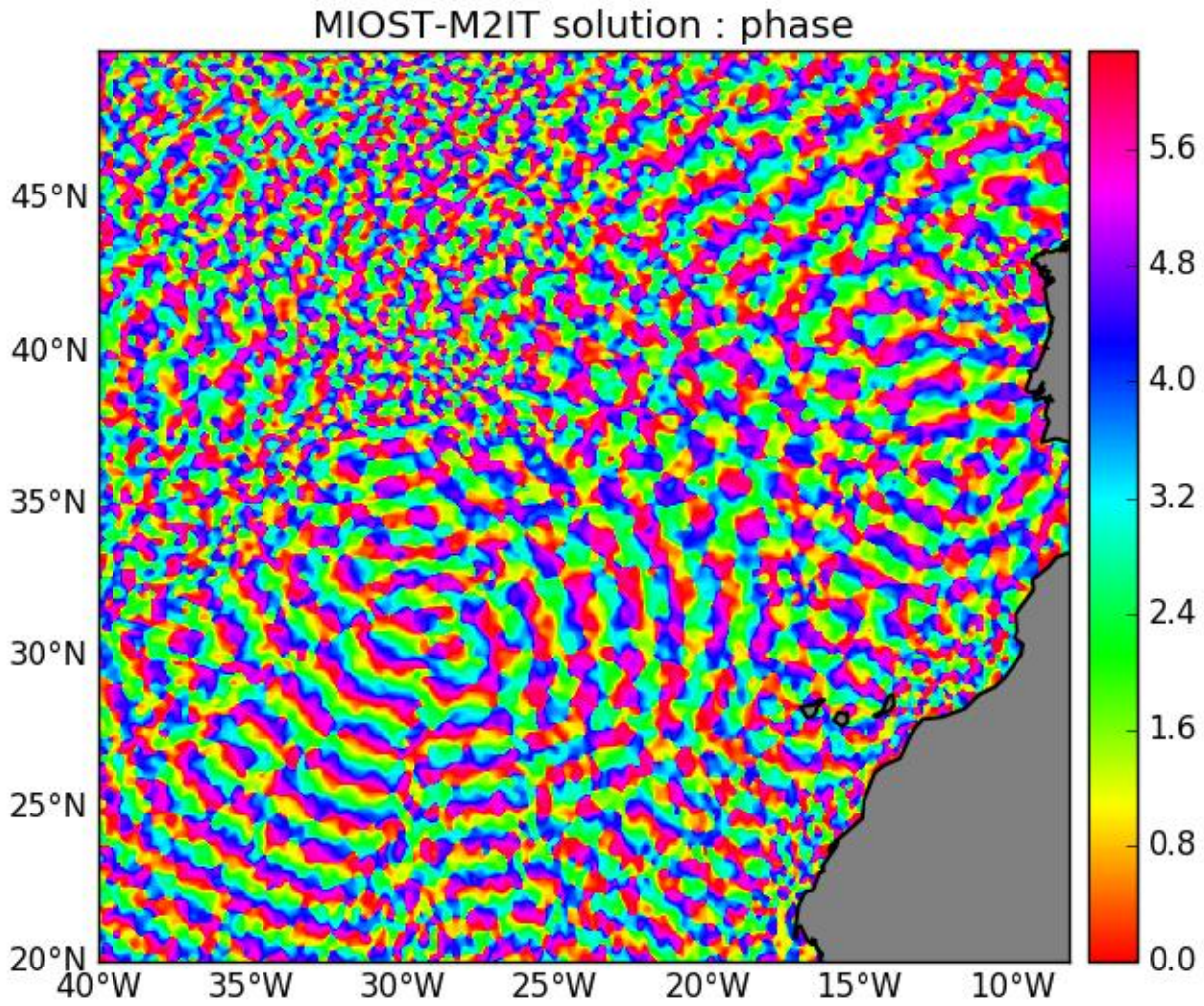


- **Overall consistency**
- More energetic than Ray or Zhao-2016 suggest
- The MIOST solution seems to reduce more variance when applied to independant data

Amplitude comparisons



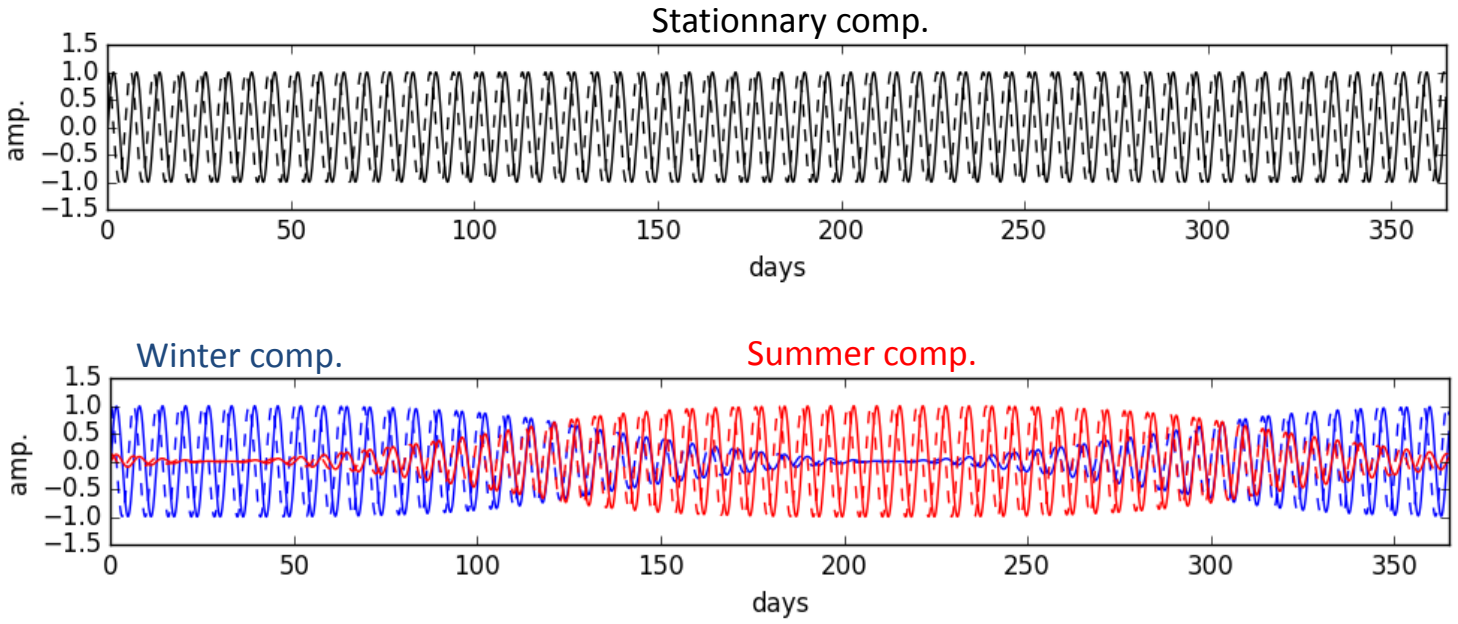
Phase comparisons



Considering non-stationary IT ?

MS-induced non-stationarity: low expectations with such purely statistical mothod (data too sparse)…
→ Modelers?

However, in regions not dominated by MS-induced unstationarity, seasonnal or climatic index modulations could be considered:



Conclusions

• This was an **attempt to inverse Mesoscales (MS) and Internal Tides (IT) in one shot**, initially performed for the need of improving MS maps.

• The **IT field seems rather consistent with existing IT models**, with a few notable differences:

→ Higher level of energy (by ~30%)

→ In the regional test, it would exhibit more variance reduction when applied to independant data

• The fields of this first test are available (email us)

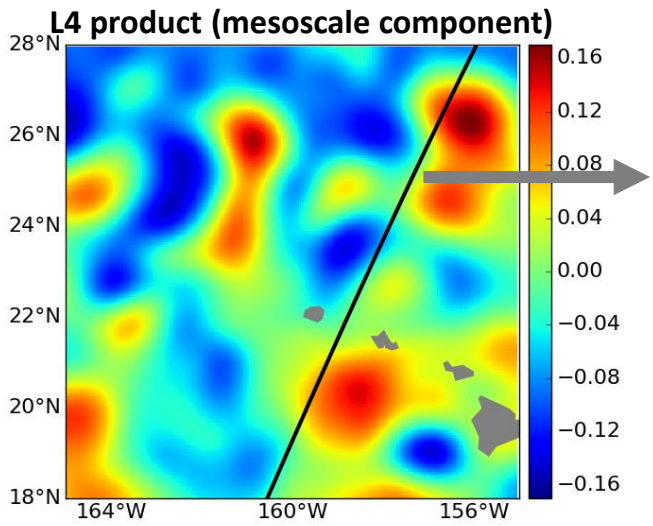
• If this solution has an interest to the tidal community, **a global run might be doable** with appropriate effort (computationally expensive : 10^{10} parameters, accurate specification of global MS/IT components, …)

<u>Further perspectives for improvements:</u>

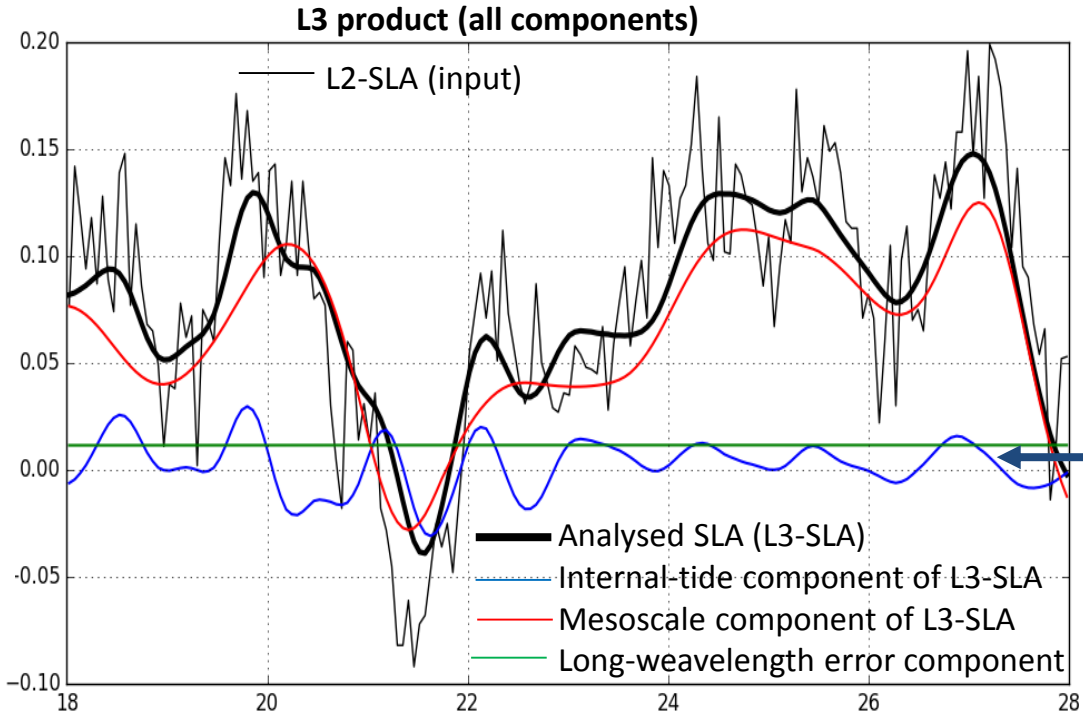
- We may add more tidal constituents
- Compatible with non-stationnary IT, not tested yet
- Activate dynamical advections of the MS components

Backup

Ainsi que les ondes internes si on inverse sur plusieurs années



O(10^7) parameters inverted to process 15 year worth of data around Hawaii

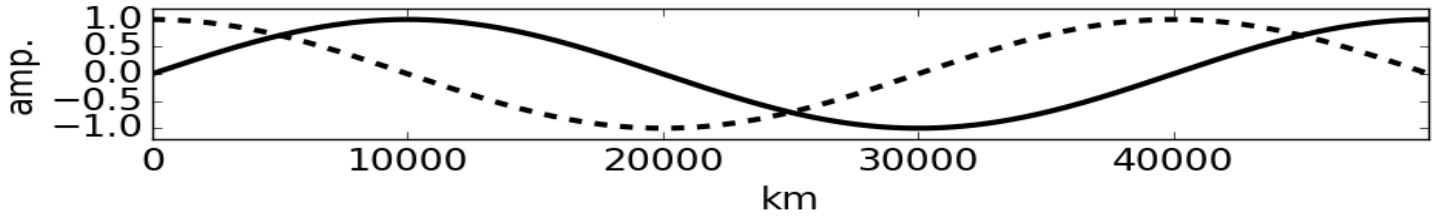


Non-zero mean: BGLO de marée capturé!

A 1D Fourier basis for long-wavelength errors

- The along-track input data feature long-wavelength signals (both errors and large-scale SSH.

- The part of this signal not coherent with large-scale patterns (e.g. a single track that pops above others) can be projected in a 1D along-track Fourier basis (discarded) to avoid mesoscale contamination.



Outline

- Context / Why considering a global Mesoscale (MS) – Internal Tide (IT) multi-year inversion?

- Implementation

- First experiment on a regional domain, over a 20-year time window

- Assessment of the IT solution

- Perspectives for solving “slowly non-stationary” IT ?

- Conclusions - Next try on the global domain?