



## Seafloor Topography and Ocean Tides from Satellite Altimetry

Ed Zaron

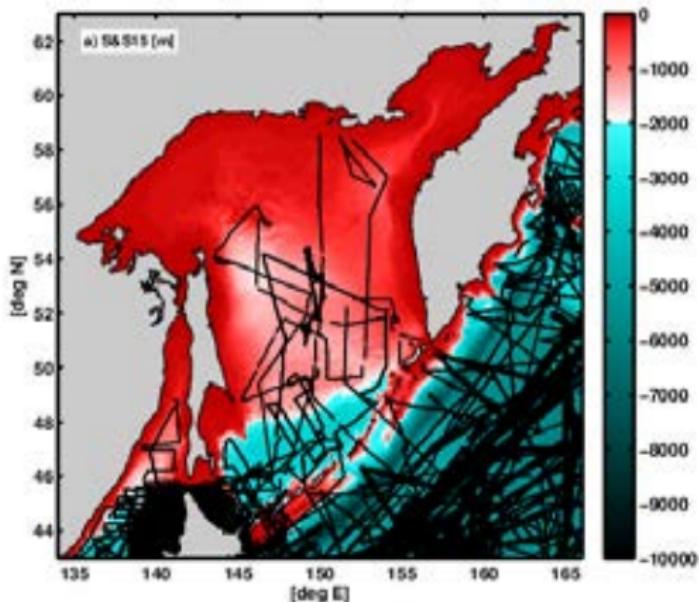
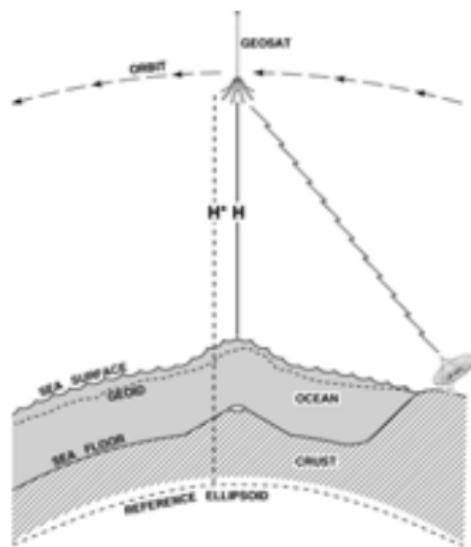
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# Outline

1. Motivation and methodology
2. Assembling the pieces:
  - topography, altimeter data, and their error models
3. An idealized identical twin experiment
4. Results for Sea of Okhotsk
5. Discussion

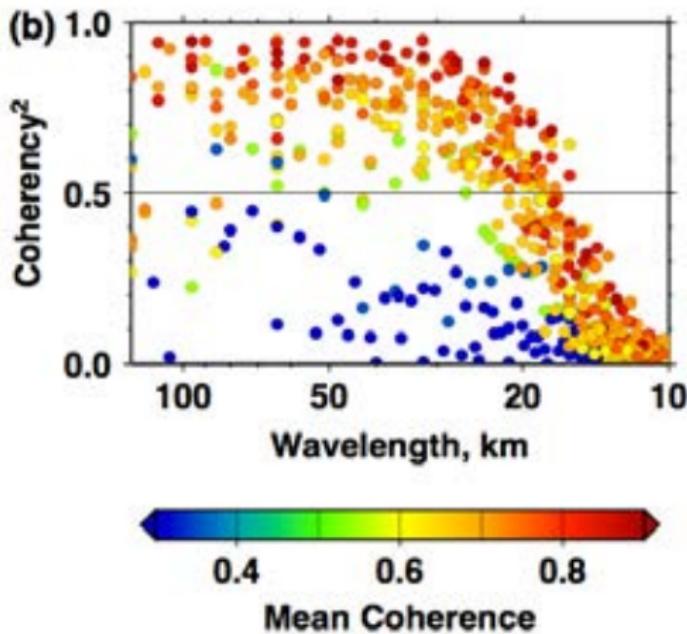
# Motivation: uncertain topography in some areas

Smith and Sandwell (JGR 1994)



- nearly global coverage
- complex relationship between topography and gravity

## Motivation: uncertain topography in some areas



Marks and Smith, Radially symmetric coherence between satellite gravity and multibeam bathymetric surveys,  
*Mar. Geophys. Res.*, 33(3), 223-227, 2012.

# Objective

To assimilate altimetric observations into a dynamical model for tides in which bottom topography is a control variable.

Precedents:

- ▶ Storm surge modeling on the European shelf (Lardner et al).
- ▶ Topography from SSH assuming large-scale dynamics (Losch and Wunsch).
- ▶ Topography from tidal currents, SSH (Hirose; Mourre et al).

## Methodology

The approach builds on OTIS, the Oregon State Tidal Inversion System, developed by Egbert and Erofeeva.

$$-i\omega U + f \times U + gH\nabla\eta + C_d|u_f|U/H = -gH\nabla\Phi^{astro} + \lambda^U \quad (1)$$

$$-i\omega\eta + \nabla \cdot U = 0 \quad (2)$$

$$H = H_0 + \lambda^H \quad (3)$$

Minimize a weighted integral of squared misfits to dynamics and data,

$$J(H, U, \eta) = \int \lambda^U C_{UU}^{-1} \lambda^U + \int \lambda^H C_{HH}^{-1} \lambda^H + \sum_{data} \frac{\epsilon^2}{\sigma^2}.$$

# Methodology

Technical developments:

- ▶ Inter-constituent coupling (same  $H$  at each tidal frequency).
- ▶ Separate representers for obs. of Re/Im parts ( $H$  is real-valued).
- ▶ New spatial covariance models for  $H$ .
- ▶ Picard iteration for nonlinearity.
- ▶ Optimizations for large multi-core machine (64 cpus, 256GB RAM).

# Where might this approach work?

Requirements for success:

- ▶ Accurate tidal SSH ( $\sigma_\eta \approx 1\%$ )
- ▶  $H$  less accurate than  $\eta$  ( $\sigma_H \approx 10\%$ )
- ▶ Accurate barotropic dynamics:
  - ▶ no baroclinic tide
  - ▶ no wetting and drying
  - ▶ bottom drag not too influential

This talk focusses on the Sea of Okhotsk, where tides are large, altimetry is plentiful, and topography is uncertain.

## Assembling the pieces: Topography

Question: What topography to start with? How to describe its spatial error covariance?

There are errors in the bathymetric data at control points. Smith (1993) and Marks and Smith (2008) note:

- ▶ positional errors, pre-GPS navigation
- ▶ sound speed errors
- ▶ blunders

In lieu of a complete error model, intercompare gridded topographies.

## Assembling the pieces: Topography

Comparison of gridded topography to JAMSTEC multibeam bathymetry [m].

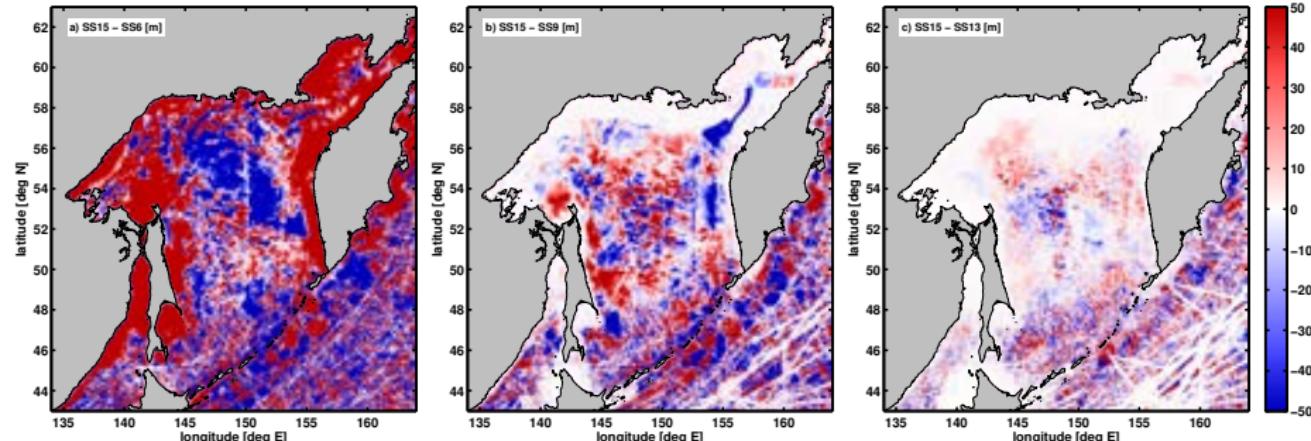
source	mean	std	mean Δ	std Δ
SS15	1079	160	0.8	4.9
SS11	1097	165	19	71
ETOPO1	1111	157	33	79
DBDB2v30	1080	168	1.7	63
GEBCO	952	111	-127	84

# Assembling the pieces: Evolution of SS over time

SSv15 - SSv6  
(1997)

SSv15 - SSv9  
(2007)

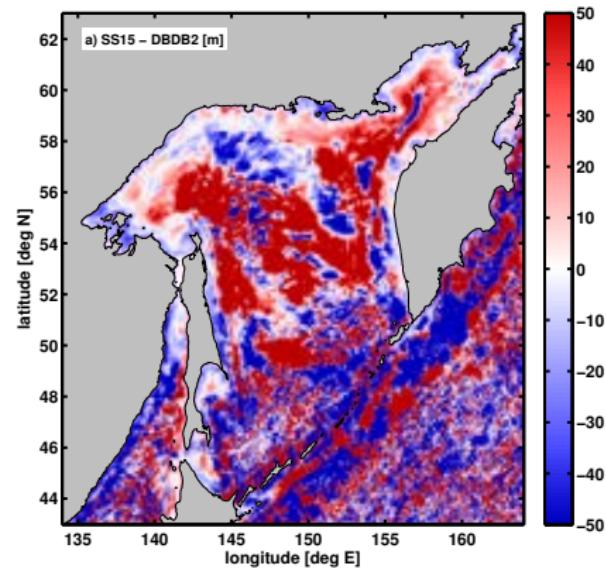
SSv15 - SSv13  
(2011)



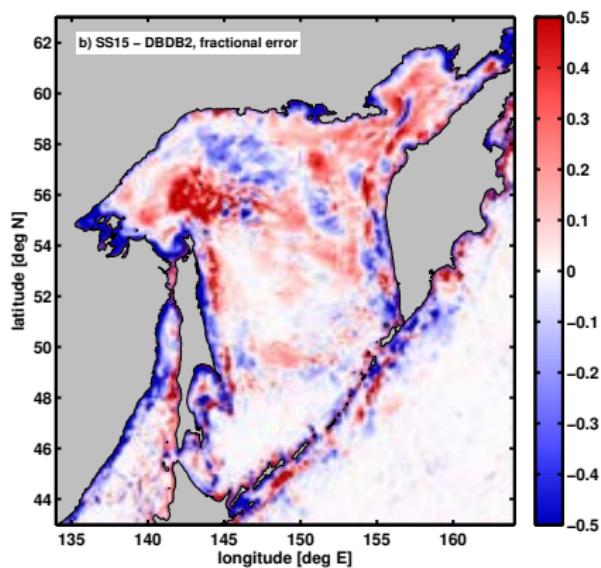
RMS differences reduced from 180m to 20m over time.

# Assembling the pieces: DBDB2 versus SSv15

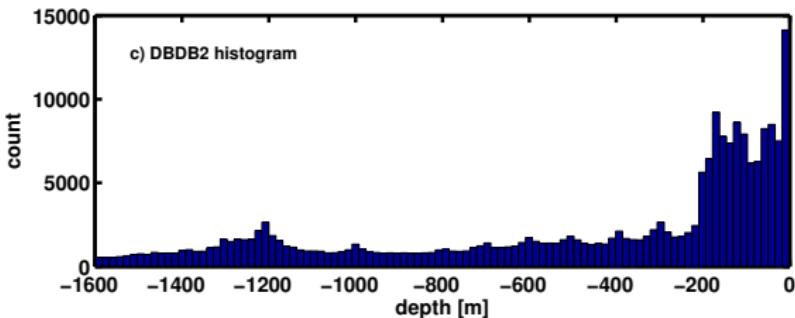
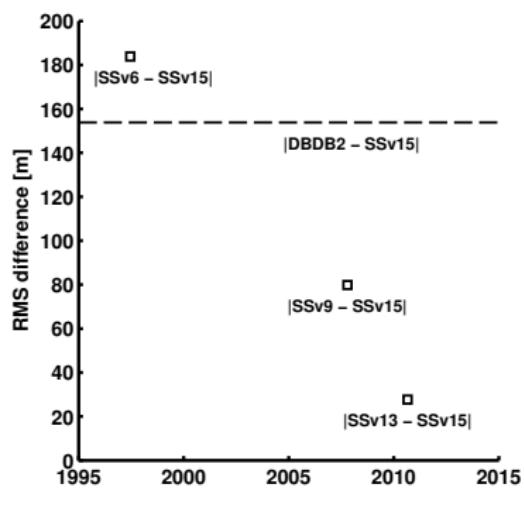
SS15 - DBDB2  
RMSD=155 m



Fractional  
Difference

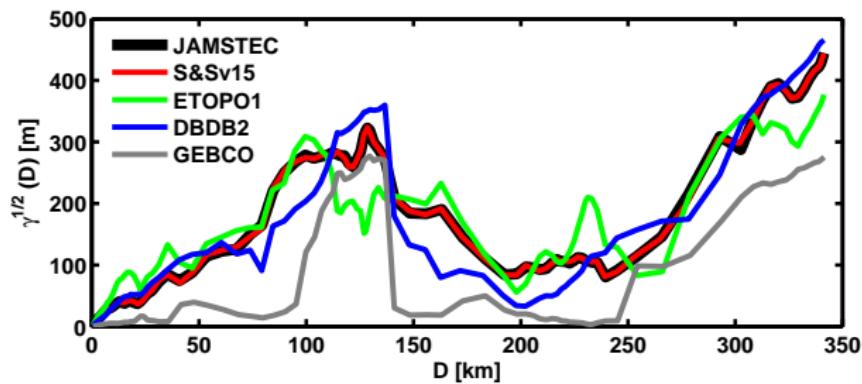


# Assembling the pieces: DBDB2 versus SSv15



# Assembling the pieces: Comparison of multiple topographies

Variogram along the cruise tracks in central Sea of Okhotsk from JAMSTEC.



# Topography: Summary

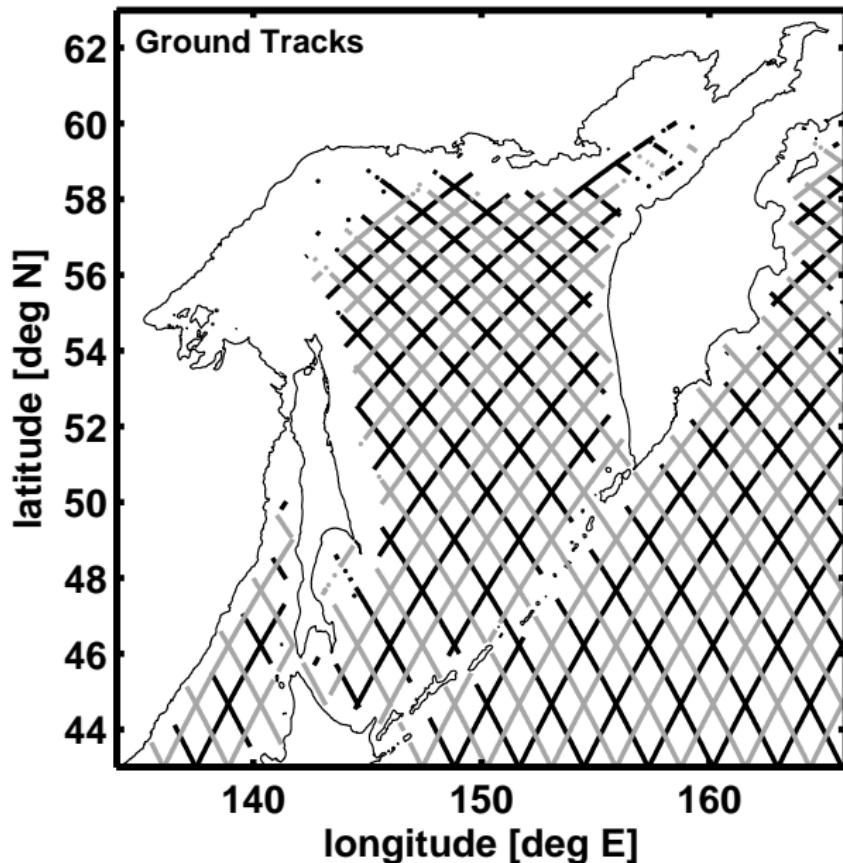
The following topographies were compared:

- ▶ Smith & Sandwell v. 6, 9, 11, 13, and 15
- ▶ GEBCO
- ▶ DBDB2v30
- ▶ ETOPO1

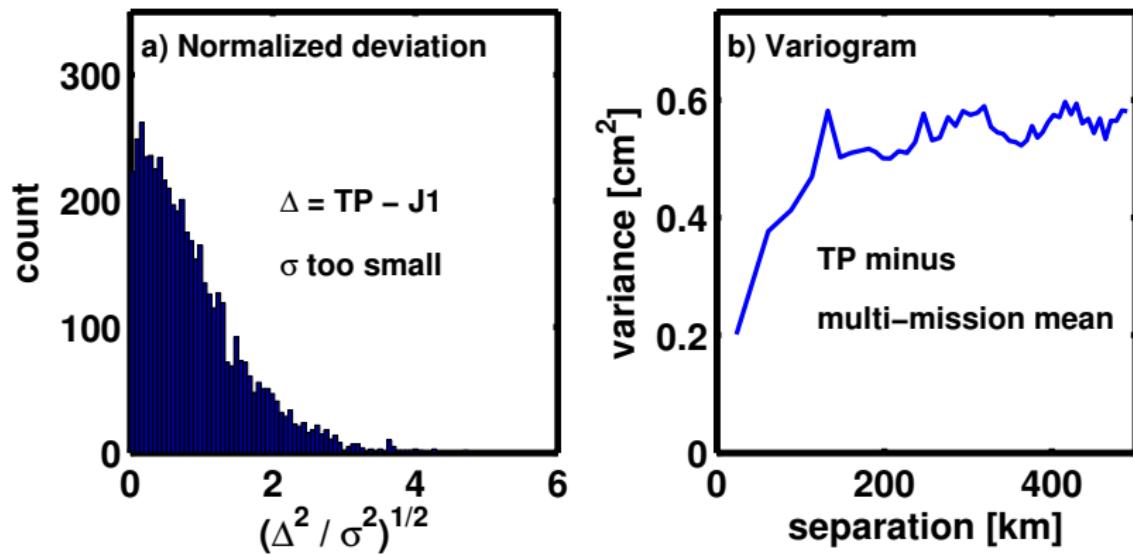
Conclusions:

- ▶ SS converging over time.
- ▶ DBDB2 and GEBCO are very smooth.
- ▶ Use ETOPO1 (hand-edited SSv9) as prior.
- ▶ Build topo. error model from DBDB2 minus SSv15.

## Assembling the pieces: Altimetry



## Assembling the pieces: Altimetry



Inflate  $\sigma$  from harmonic analysis by 30%.

Use data from cross-overs.

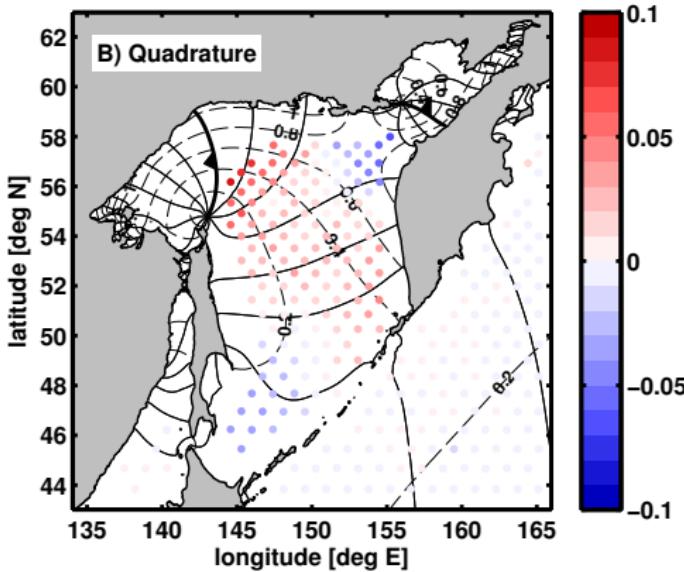
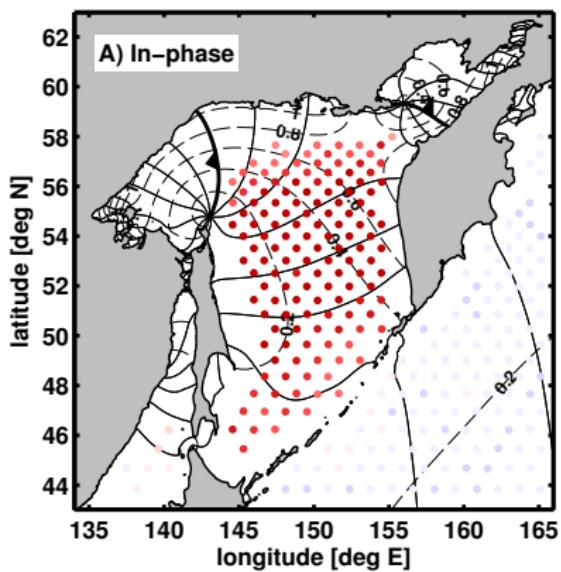
## Application to the Sea of Okhotsk

- ▶ Approx. 3km resolution
- ▶  $M_2$ ,  $S_2$ ,  $K_1$ , and  $O_1$
- ▶ Quasi-linear drag,  $C_d |u_f| U/H$ , following Snyder et al (1979)
- ▶ Open boundary conditions from TPXO7.2-ATLAS

Tide Model	RMSE [cm]
TPXO7.2-Atlas	1.3
DBDB2	12
ETOPO1 ( $C_d = 1.5 \times 10^{-3}$ )	5.4
ETOPO1 ( $C_d = 3.0 \times 10^{-3}$ )	6.2

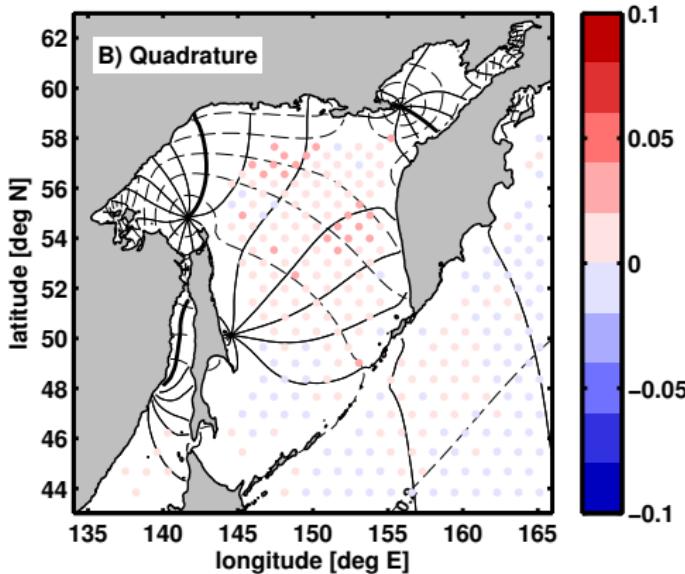
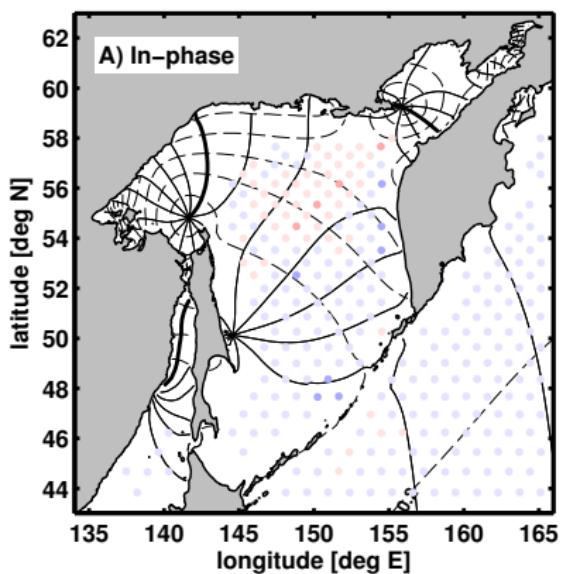
# Application to the Sea of Okhotsk

Prior model has large error.



# Application to the Sea of Okhotsk

TPXO7.2 fits data within nominal uncertainty.



## An identical twin experiment

Purpose: Demonstrate the importance of the error model for the topography.

The topography correction,  $\lambda^H$ , is computed from a spatial error covariance model and an adjoint sensitivity, but the latter depends on the  $\eta$  and  $U$  fields:

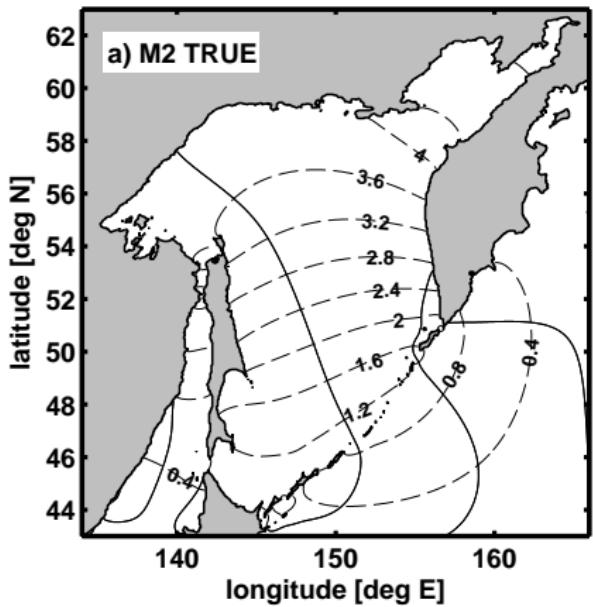
$$\lambda^H(x) = \int C_{HH}(x, y) \left( -g\boldsymbol{\mu} \cdot \nabla \eta + C_d |u_f| \frac{\boldsymbol{U} \cdot \boldsymbol{\mu}}{H^2} \right) dy \quad (4)$$

Adjoint sensitivity is dominated by regions where  $\nabla \eta$  is large.

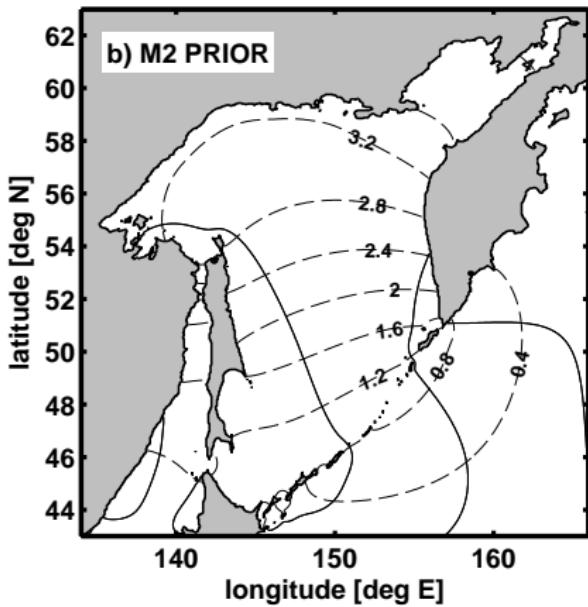
(Notation:  $x = (\theta, \phi)$  is lat.-lon. coordinate)

# An identical twin experiment

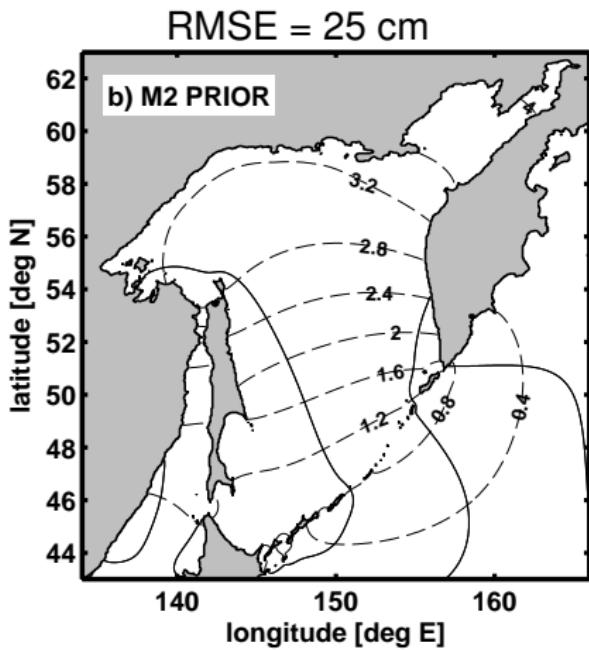
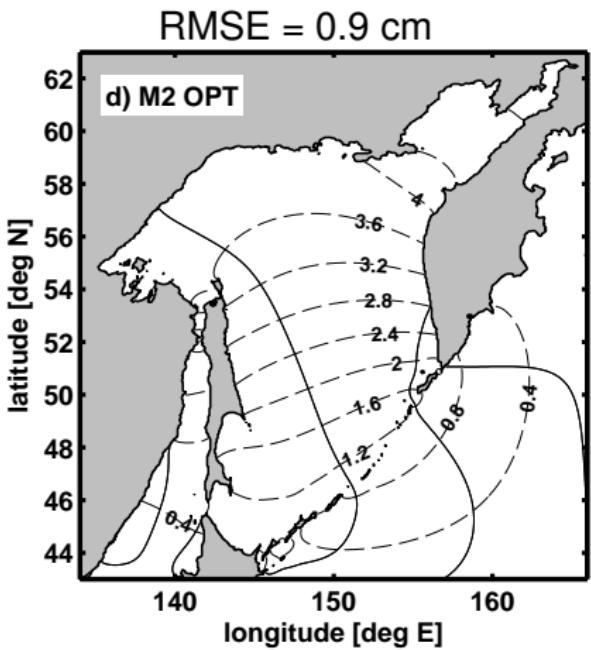
RMSE = n/a



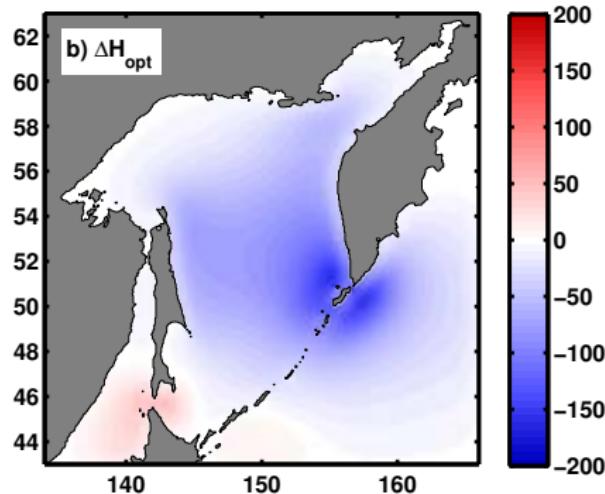
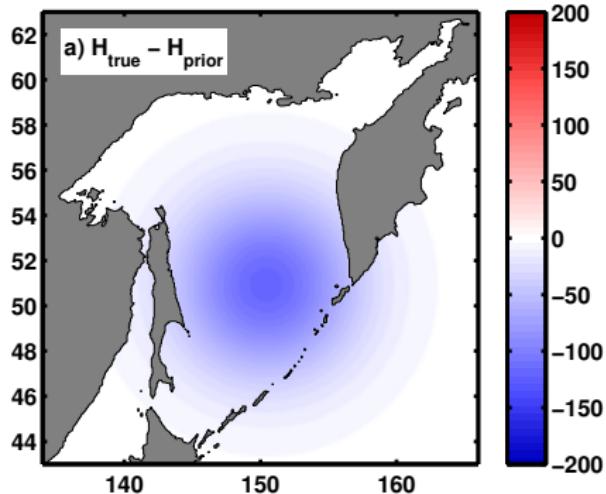
RMSE = 25 cm



# An identical twin experiment



# An identical twin experiment

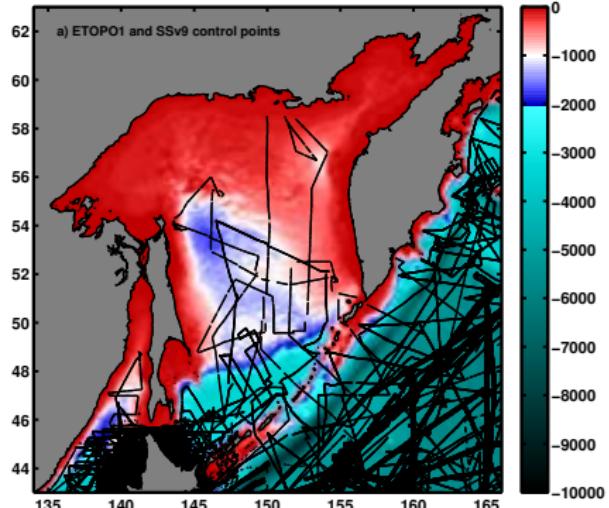


# Development of Spatial Error Covariance Model

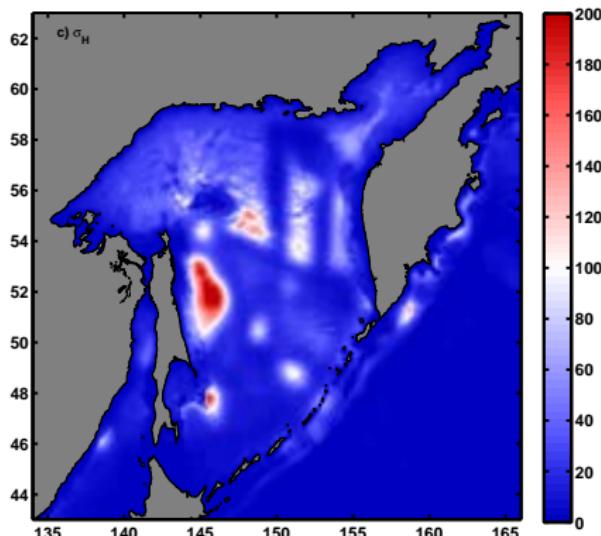
Factorize  $C_{HH}$  in terms of a variance and correlation:

$$C_{HH}(x, y) = \sigma_H(x)c_H(x, y)\sigma_H(y) \quad (5)$$

ETOPO1

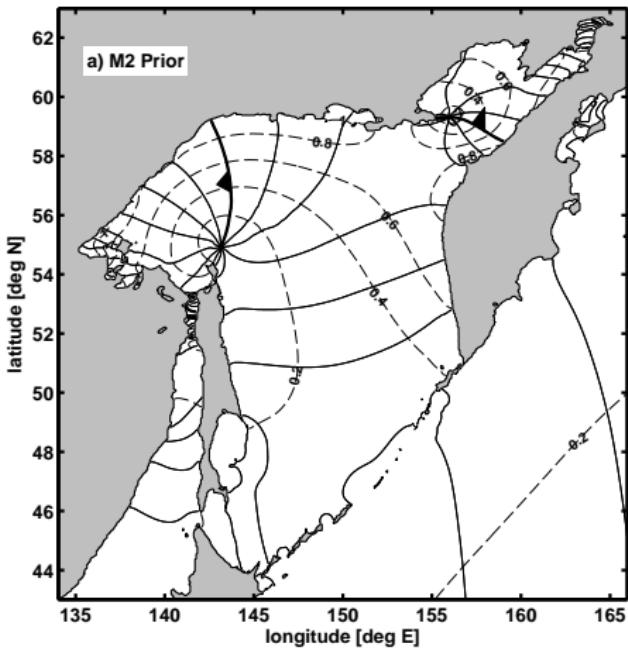


$\sigma_H$



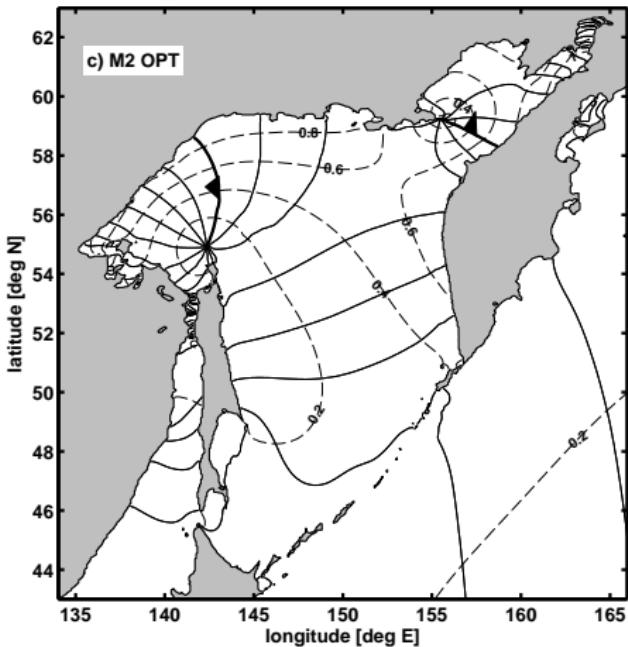
# Sea of Okhotsk (ETOPO1 + altimetry)

Prior: 4.1cm rmse



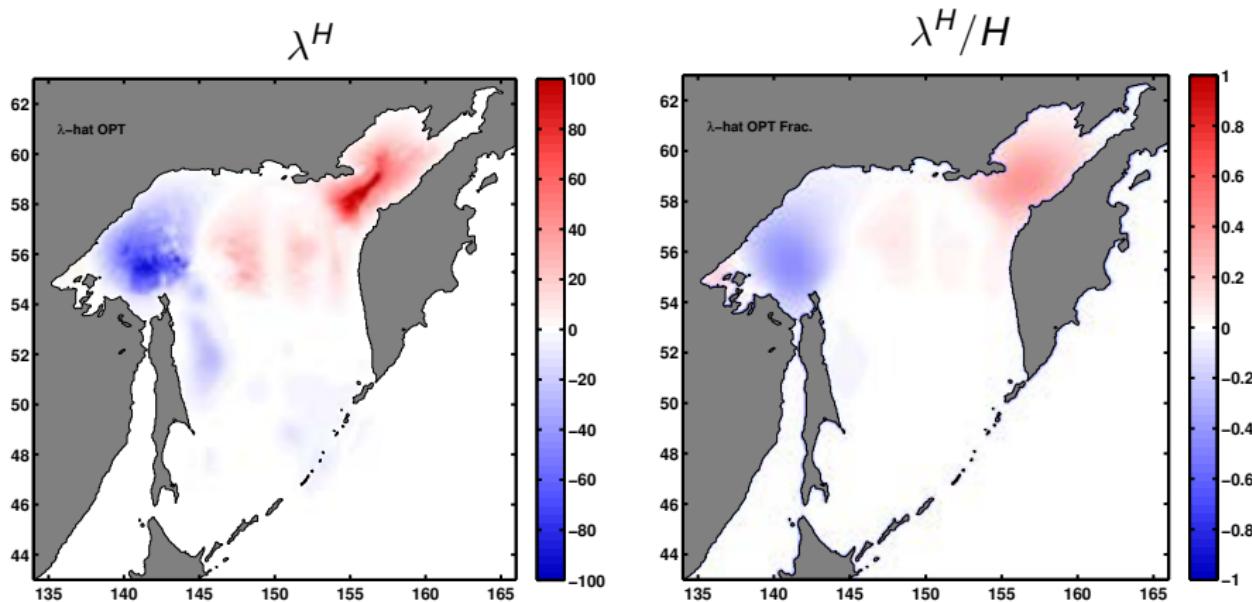
# Sea of Okhotsk (ETOPO1 + altimetry)

Optimal: 2.8cm rmse



Topographic adjustments explain about 50% of the SSH error variance.

# Sea of Okhotsk (ETOPO1 + altimetry)



## Conclusions & Further Questions

### Summary:

- ▶ Relatively small corrections to bottom topography can explain a significant fraction of the discrepancy between observed and modeled tides.
- ▶ The topographic inversion problem is strongly nonlinear.
- ▶ Spatial structure of errors in gridded topographies is not well characterized.
- ▶ The estimated topography depends strongly on the assumed spatial covariance,  $C_{HH}$ .

### Questions:

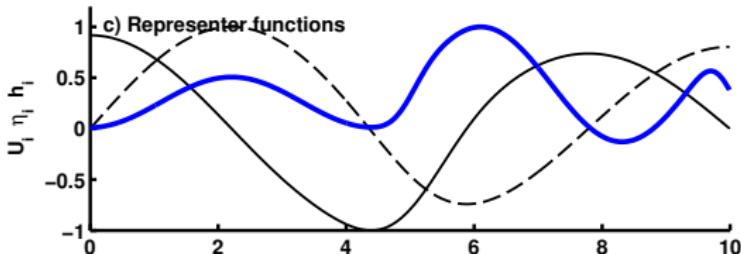
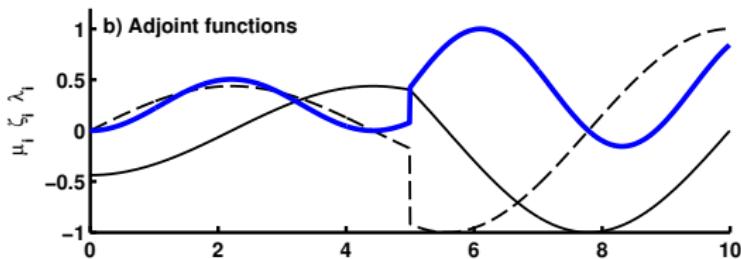
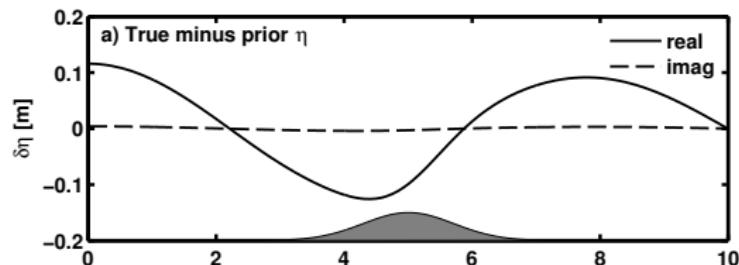
- ▶ 1-d idealized experiments show importance of small-scale roughness for de-tuning resonance. What are implications for the first guess?
- ▶ Where are other good sites for application?

## Backup Slides

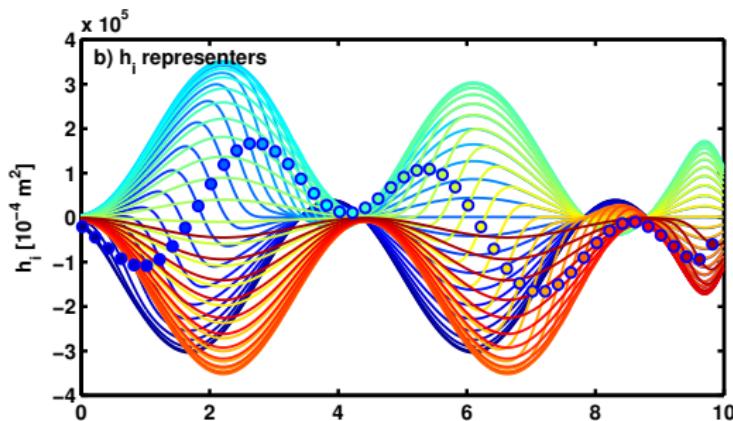
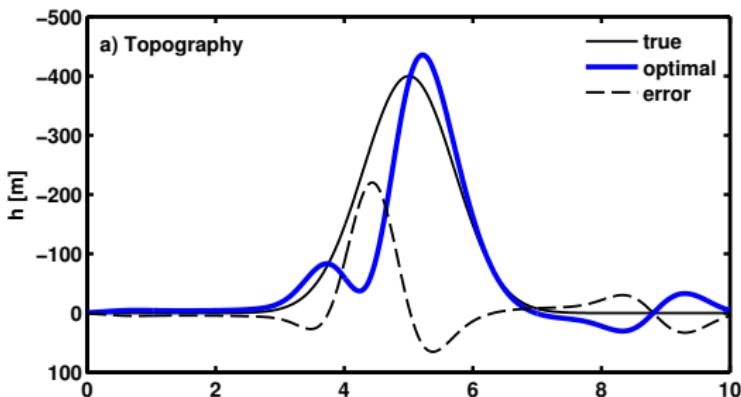
Observability in a 1-d idealized model:

- ▶ Nonlinear dependence on first guess.
- ▶ No corrections where  $|\nabla \eta| = 0$ .
- ▶ → multiple constituents are required.
- ▶ Realizability condition:  $\lim \sigma_H/H \rightarrow \text{const.}$  as  $H \rightarrow 0$ .
- ▶ Relative  $H$  error is 5 to 10× relative  $\eta$  error.

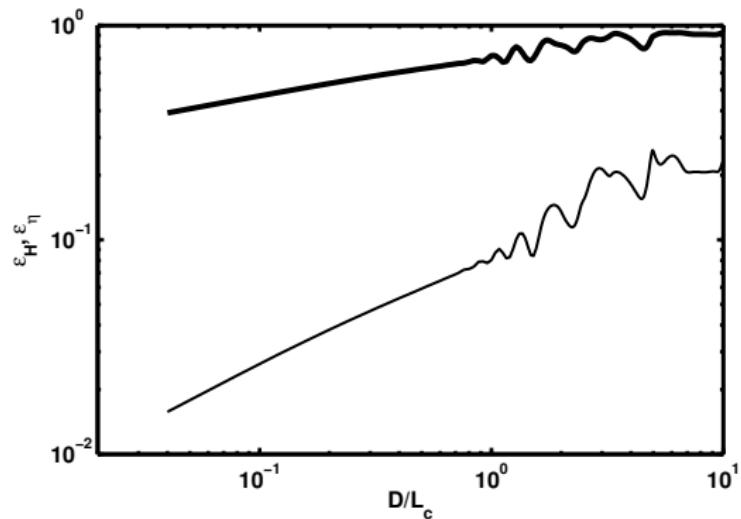
# 1-d Experiment: isolated bump



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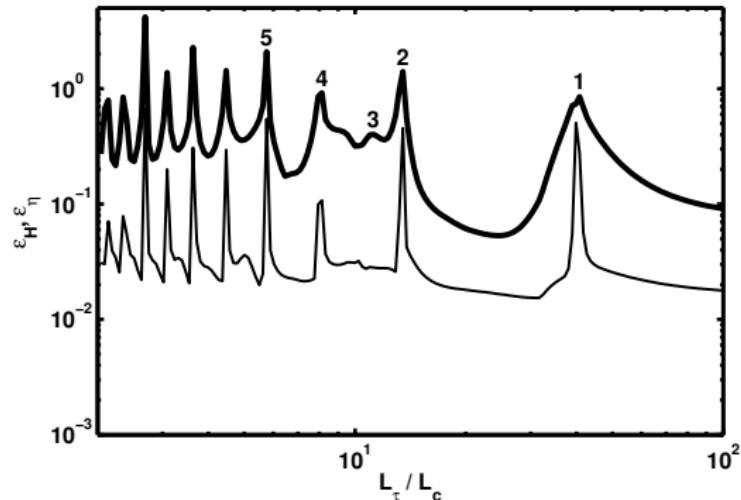
## 1-d Experiment: isolated bump



$\epsilon_H$  and  $\epsilon_\eta$  are relative errors in  $H$  and  $\eta$  as estimated by data assimilation.

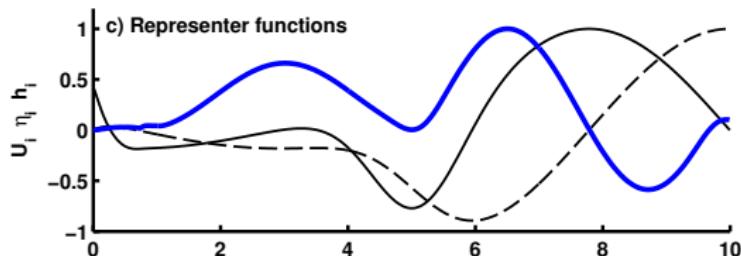
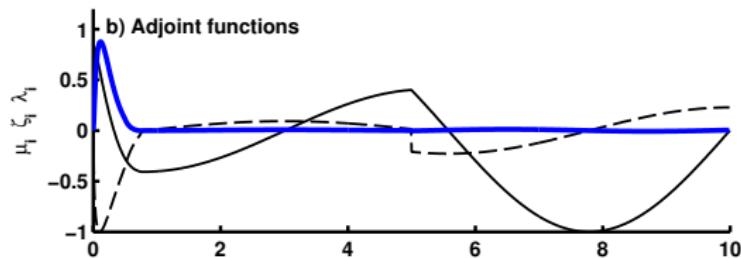
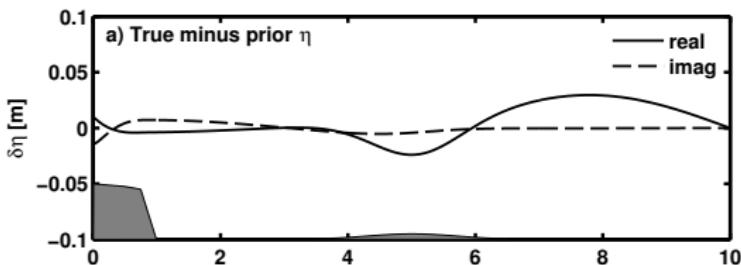
Estimation error decreases as data spacing  $D$  goes to zero.  $L_c$  is the half-width of the topographic bump.

# 1-d Experiment: isolated bump

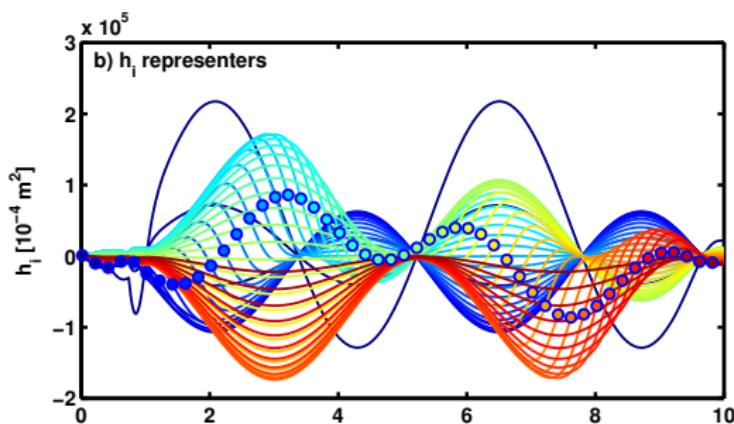
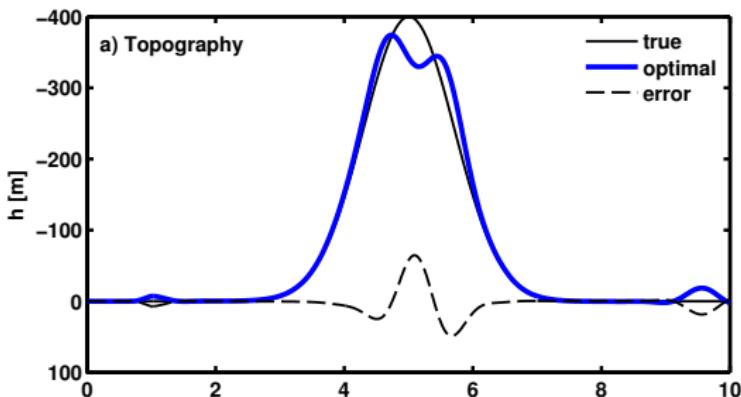


Estimation error varies strongly with tidal wavelength. Errors are worst at near-resonance.

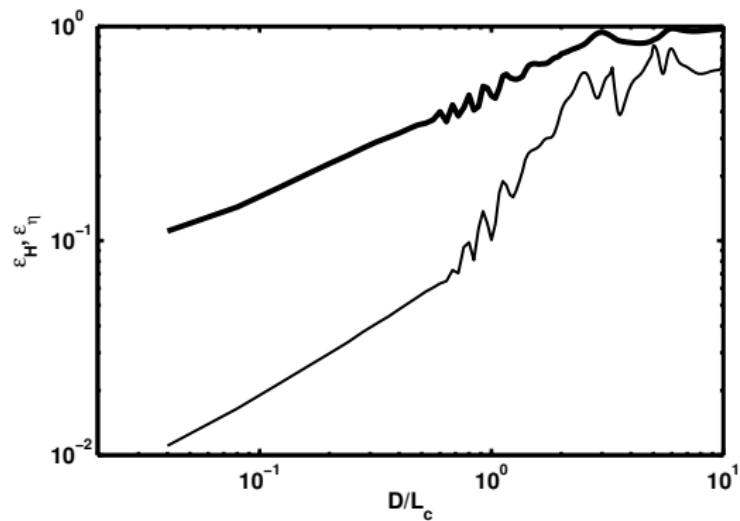
# 1-d Experiment: shelf + isolated bump



# 1-d Experiment: shelf + isolated bump

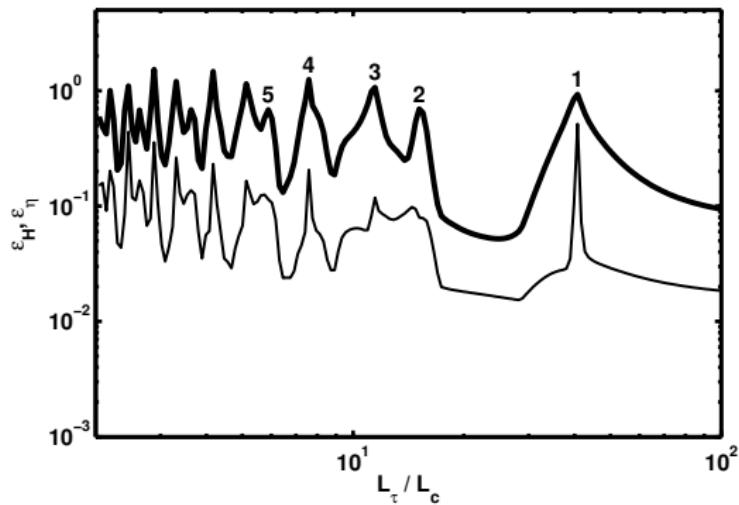


# 1-d Experiment: shelf + isolated bump

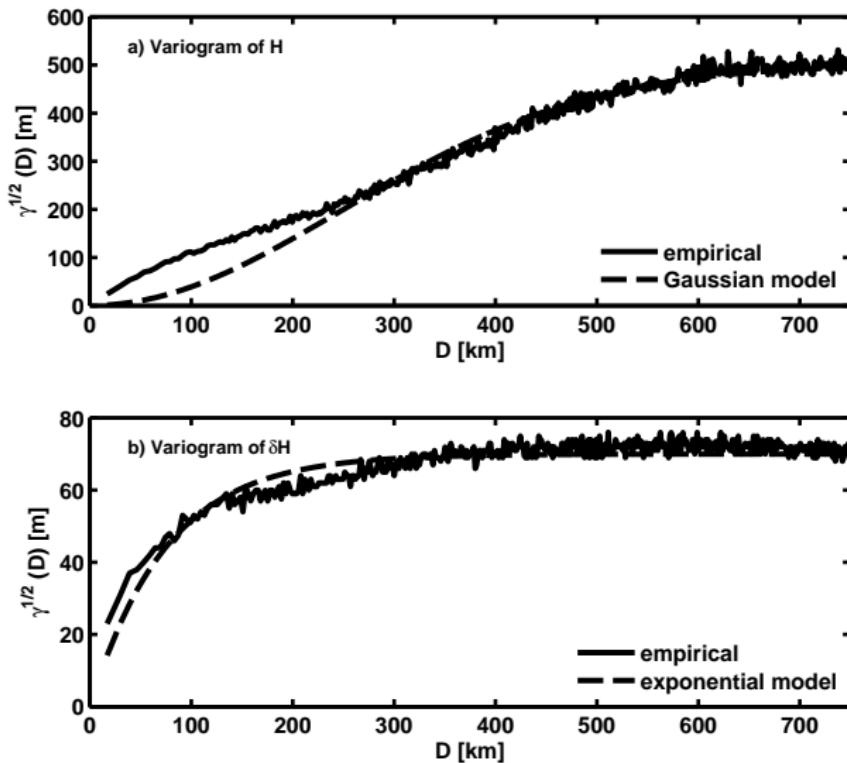


Note:  $\sigma_H(x) = \alpha H(x)^p$ , with  $p > 1$  is required in order to suppress the sensitivity on the shelf.

# 1-d Experiment: shelf + isolated bump



# Error Model for Topography



Topography "error":  $\delta H = \text{ETOPO1} - \text{DBDB2}$