2D/3D tidal modelling: T-UGOm spectral solver

F. Lyard, D. Allain, C. Nguyen, Y. Soufflet

LEGOS/CNRS, Toulouse, France

Contact: florent.lyard@legos.obs-mip.fr

T-UGOm is basically a time-stepping model… so

Why doing frequency-domain modeling ?

- First implemented to downscale global tidal atlases at open boundaries (currents issue)

- Realistic solutions
  - true turbulence closure
  - non-linearities solved by iterating solving cycle

- Extremely cheap compared to time-stepping modeling

- Easy implementation of various experiments/tuning allowed

- No separation issue (seasonal variability easier to examine)

- Well suited for frequency-domain data assimilation

Complementary to time-stepping simulation efforts…

Spectral equations for astronomical tides

(equations for non-linear tides also available)

- Quasi-linearised, spectral Shallow-Water equations
  - Momentum equation $j\omega\mathbf{u} + f \times \mathbf{u} = -g\nabla(\eta + \delta) + g\nabla\Pi - \mathbf{F}\mathbf{u} - \mathbf{D}\mathbf{u}$
  - continuity equation $j\omega\eta + \nabla \cdot h\mathbf{u} = 0$

$$\mathbf{F} = \begin{bmatrix} r & r' \\ r'' & r''' \end{bmatrix} \qquad \mathbf{D} = c\rho_0 \frac{\kappa^{-1}}{\omega} \left[\left(N^2 - \omega^2 \right) \left(\omega^2 - f^2 \right) \right]^{\frac{1}{2}} \left[\nabla h \cdot^t \nabla h \right]$$

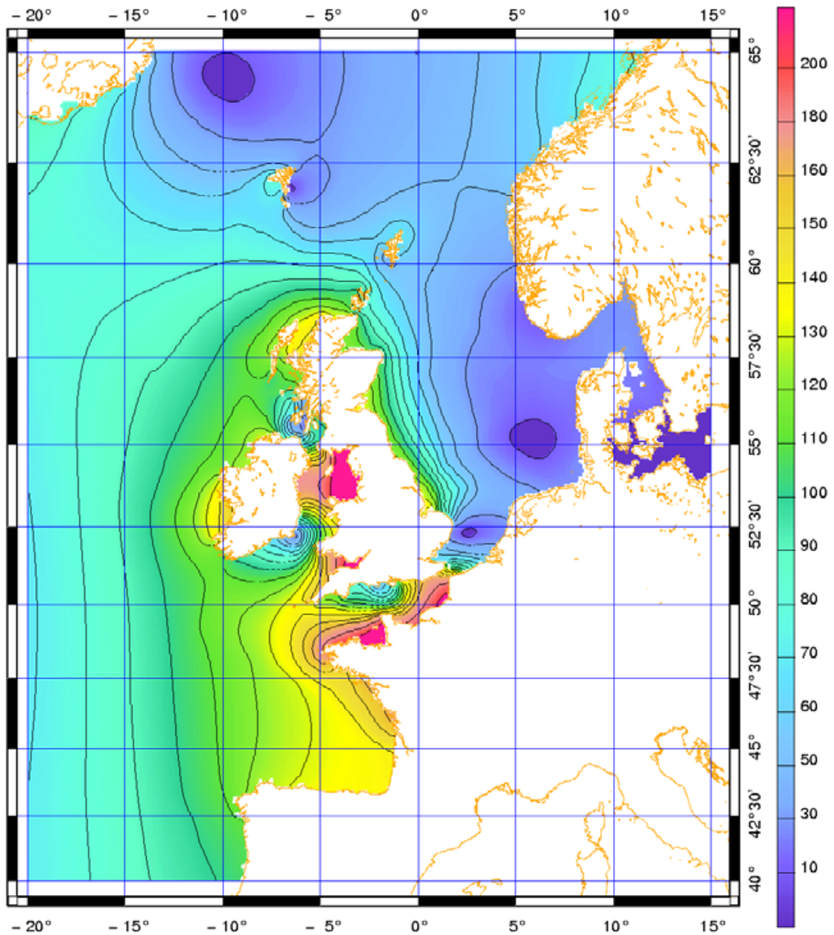
see Lyard et al., 2006, Modelling the global ocean tides: modern insights from FES2004, Ocean Dynamics http://dx.doi.org/10.1007/s10236-006-0086-x

- Discrete equations (transport)
  - Momentum equation $\mathbf{MU} = -\mathbf{G}\eta + \ni$
  - Wave equation $j\omega\mathbf{B}\eta - \nabla \cdot \mathbf{M}^{-1}\mathbf{G}\eta = -\mathbf{D}\mathbf{M}^{-1}\mathbf{G}\eta = -\nabla \cdot \mathbf{M}^{-1}\ni$

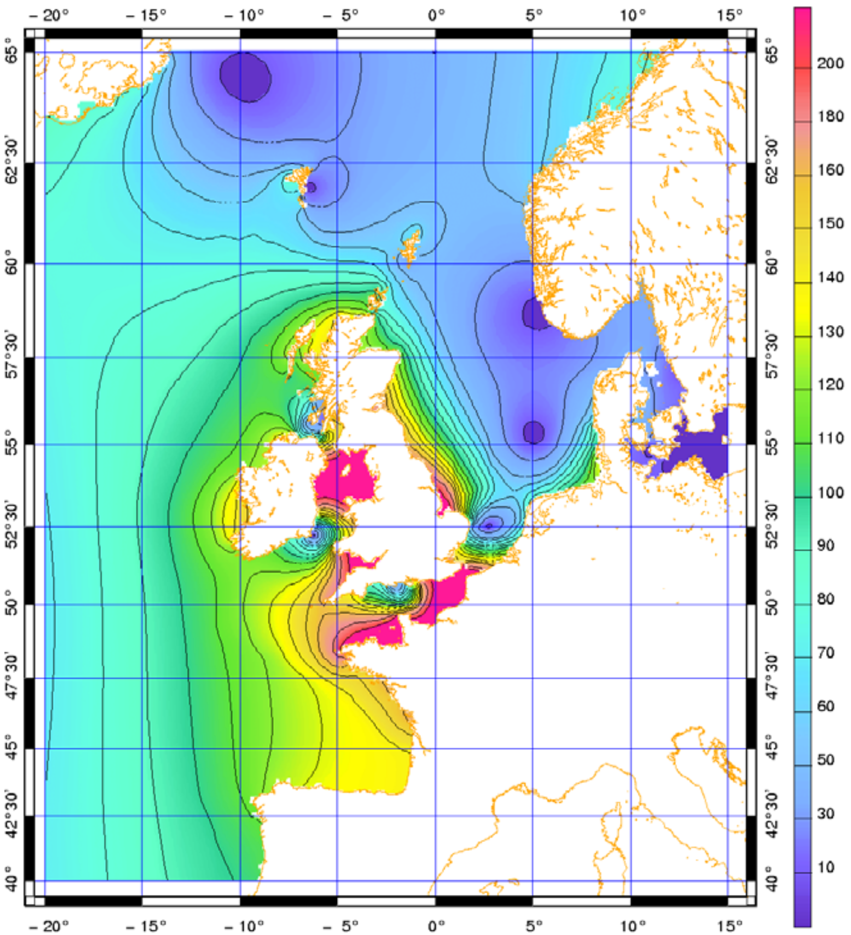
  discrete velocity and elevation solution are fully consistent

- Double complex **sparse** matrix solver: PASTIX
  - OpenMP optimized, MPI available
  - Stable for large numbers of DoF (UMFPACK is not)

non-linearities: solved by iterating solving cycle



M2 elevation : iteration 1

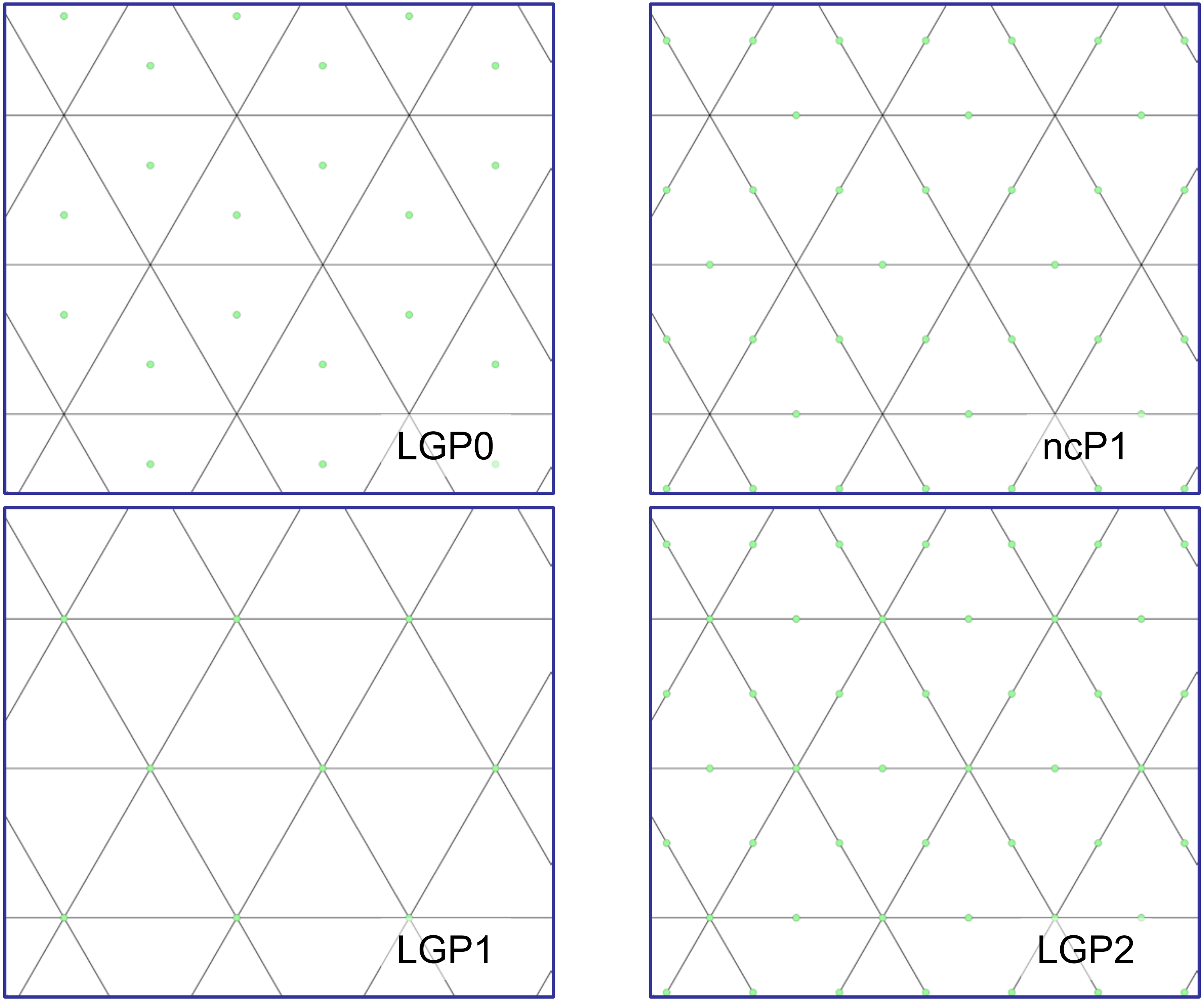


M2 elevation : iteration 10

Triangle elements

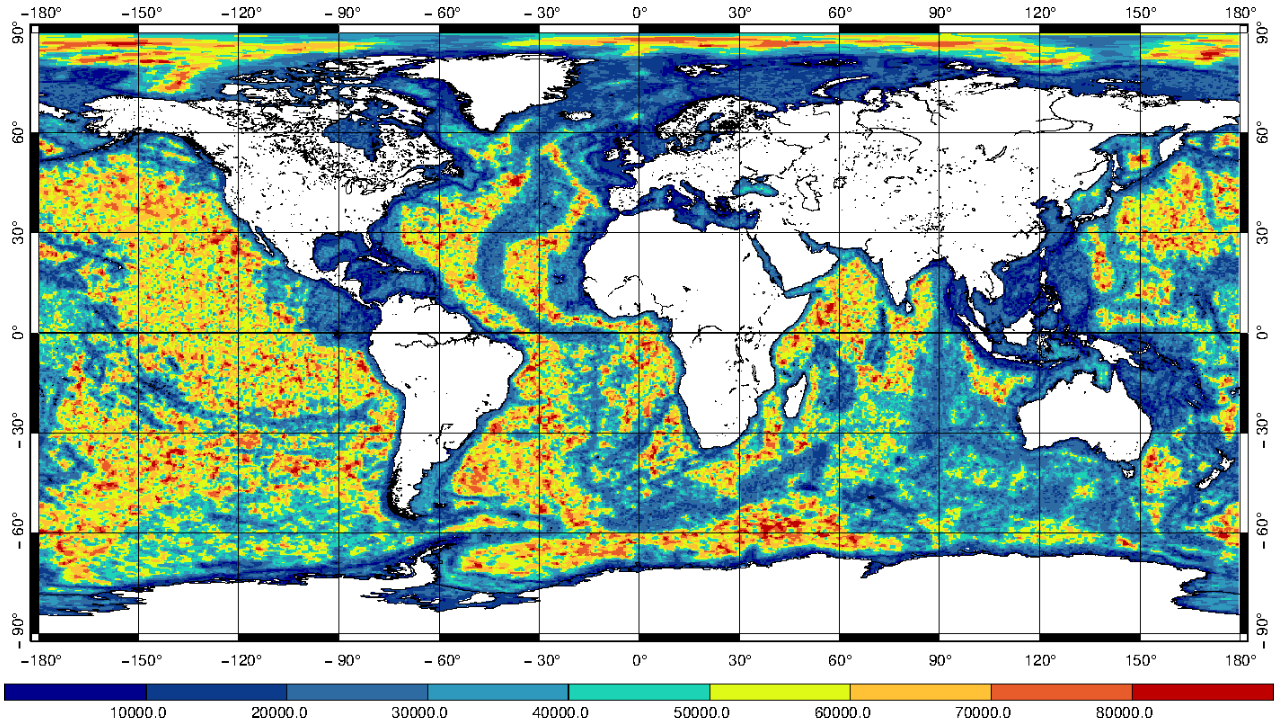
2D spectral modeling

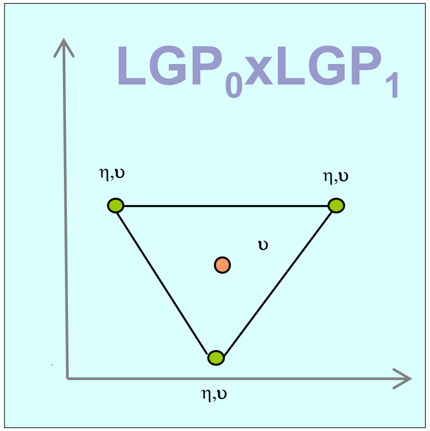
Available discretisation

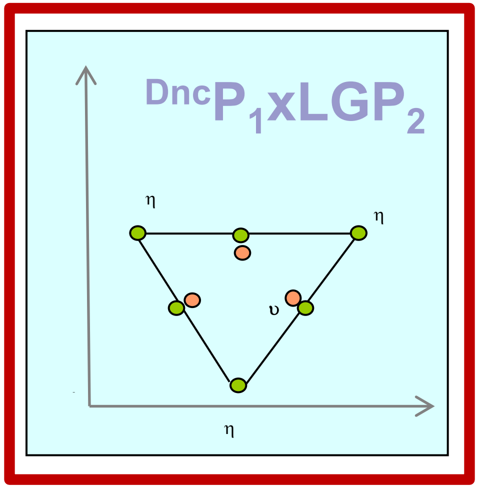


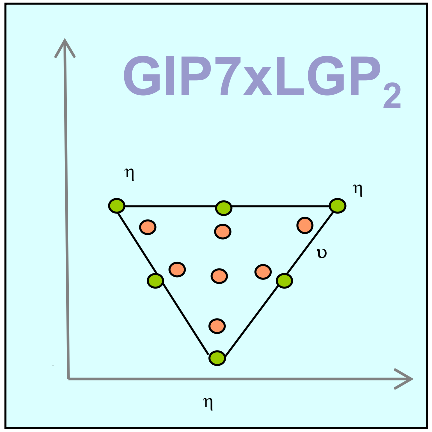
FES2014 discretisation, resolution, DoF

- **750 000 vertices**
- **1 500 000 triangles**
- **3 000 000 elevation nodes**
- **4 500 000 velocity nodes**









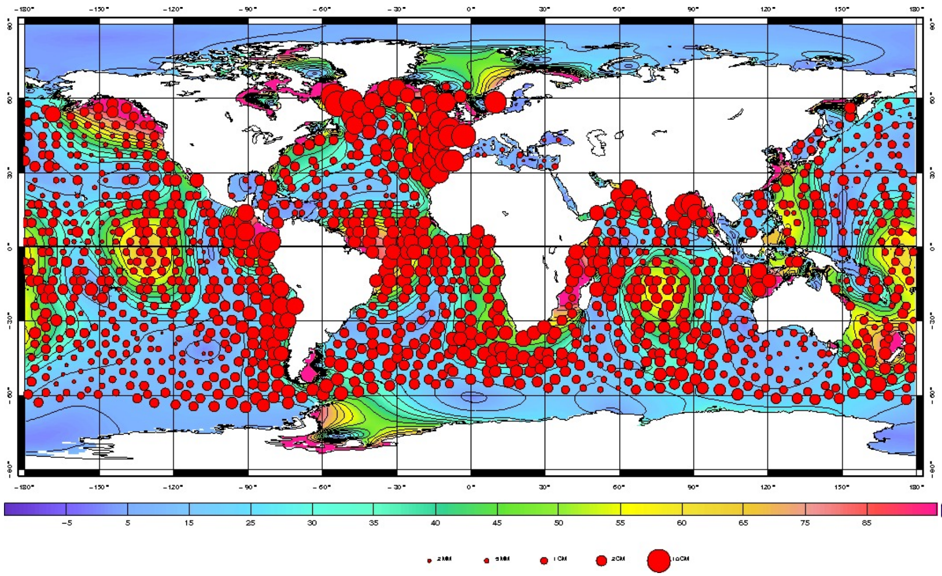
2D global ocean tides and storm surges modeling

Objectives in preparation of SWOT mission

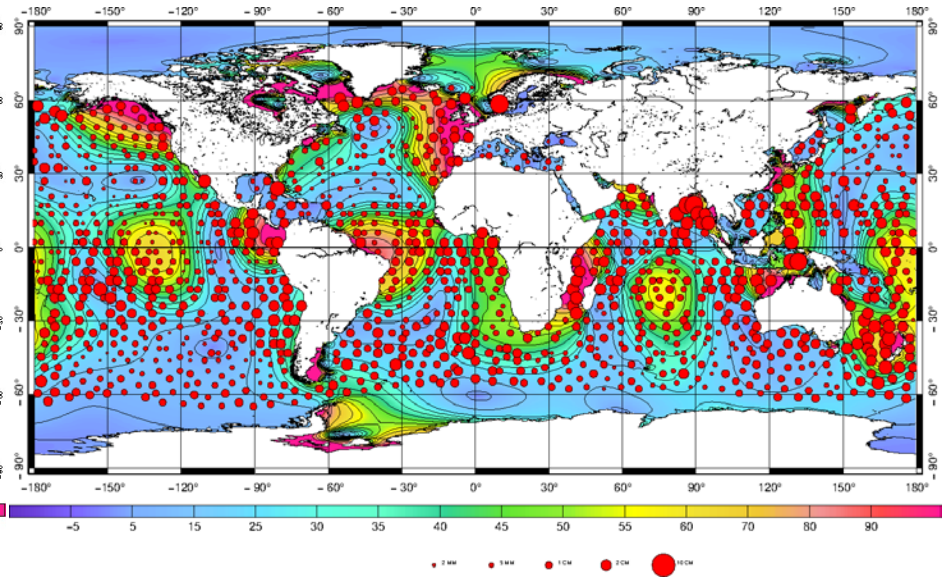
- Continuously improve hydrodynamic tidal solution accuracy
- Use data assimilation (tide gauge and altimetry-derived observations) to reach altimetry requirements
- Upgrade the global ocean storm surges simulator
- Increase coastal resolution

wave	FES2012	FES2014
M2	24 / 93 mm	**13 / 53 mm**
S2	10 / 28 mm	**8 / 18 mm**
K1	11 / 30 mm	**9 / 23 mm**
O1	12 / 30 mm	**7 / 20 mm**

comparisons against TP-J1-J2 (deep / shelf)

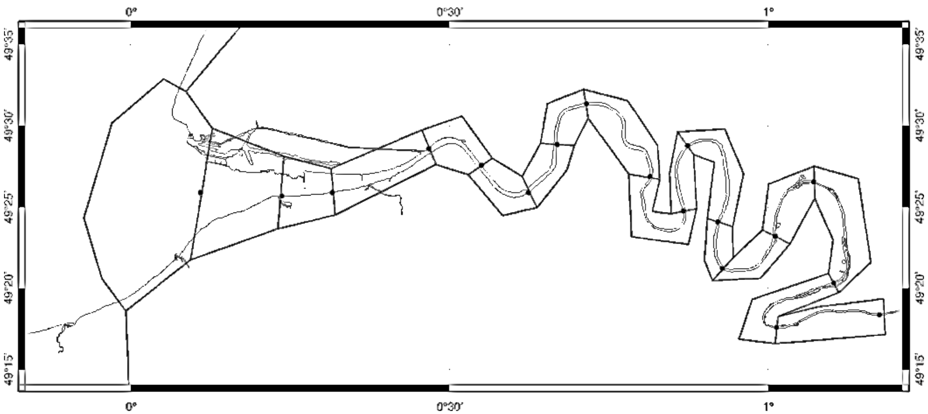


FES2012 hydrodynamic reference

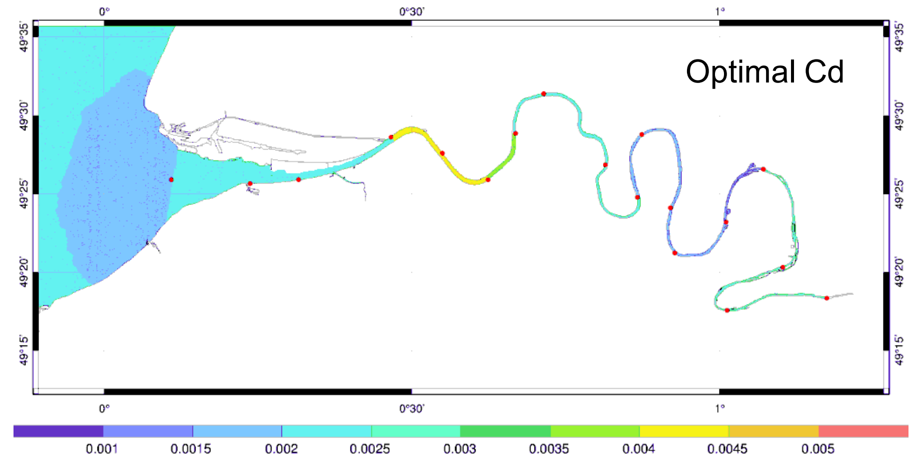


FES2014 hydrodynamic reference

Spectral estuarine simulation



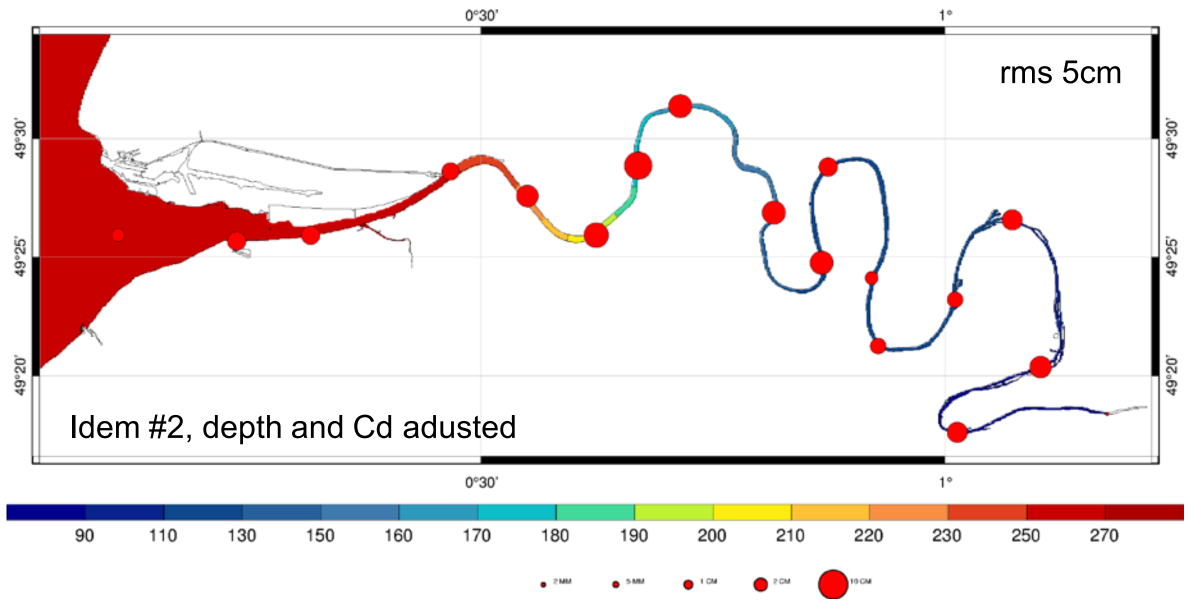
Calibration polygons (Cd and H offset)



Following damping and phase lags, Cd and H are modified in sections until fitting observations

So calibration works:

- Spectral calibration much quicker
- Still, automatized calibration schemes would be nice



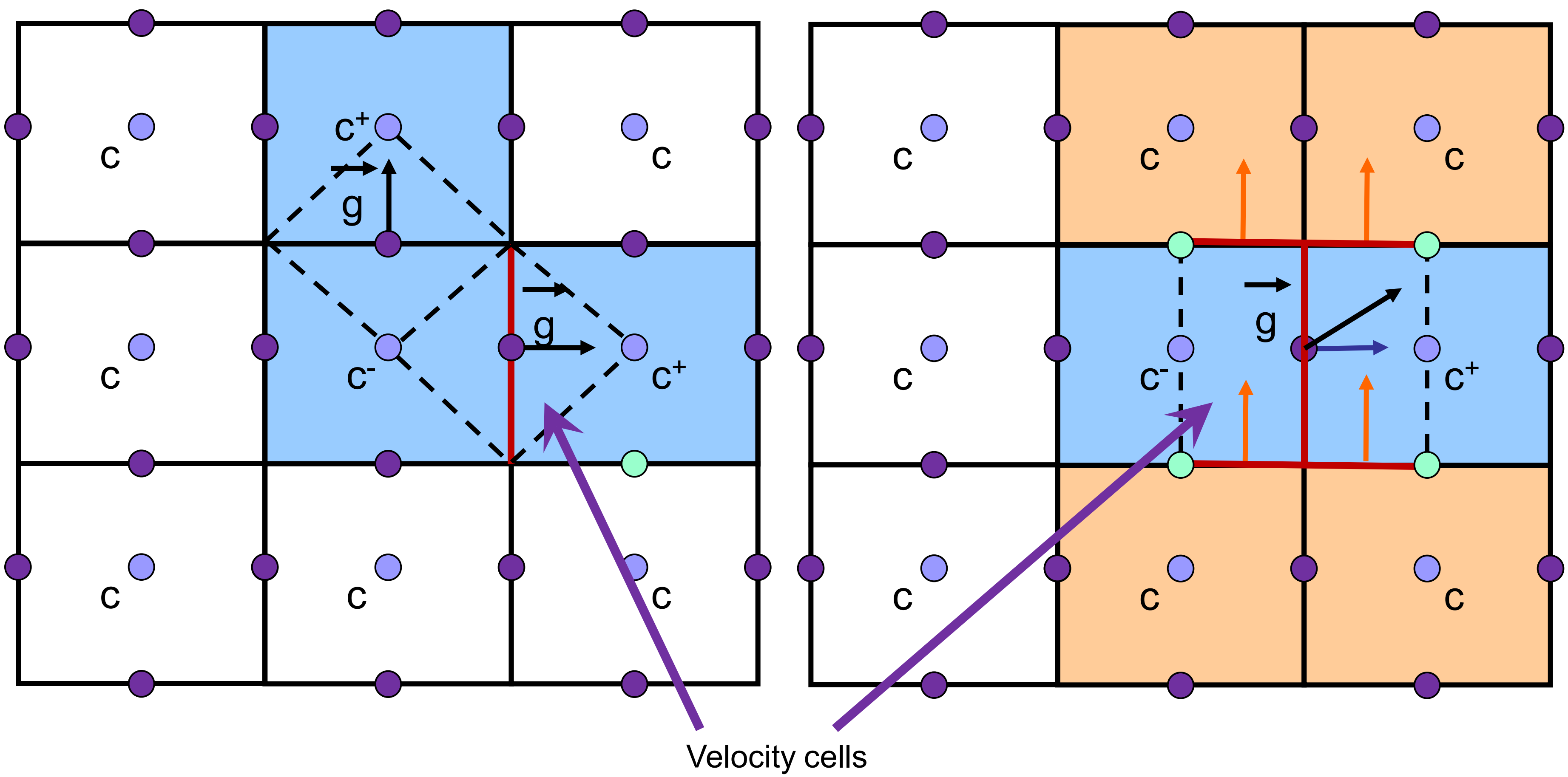
Quadrangle elements
2D spectral modeling

Pressure gradient issue

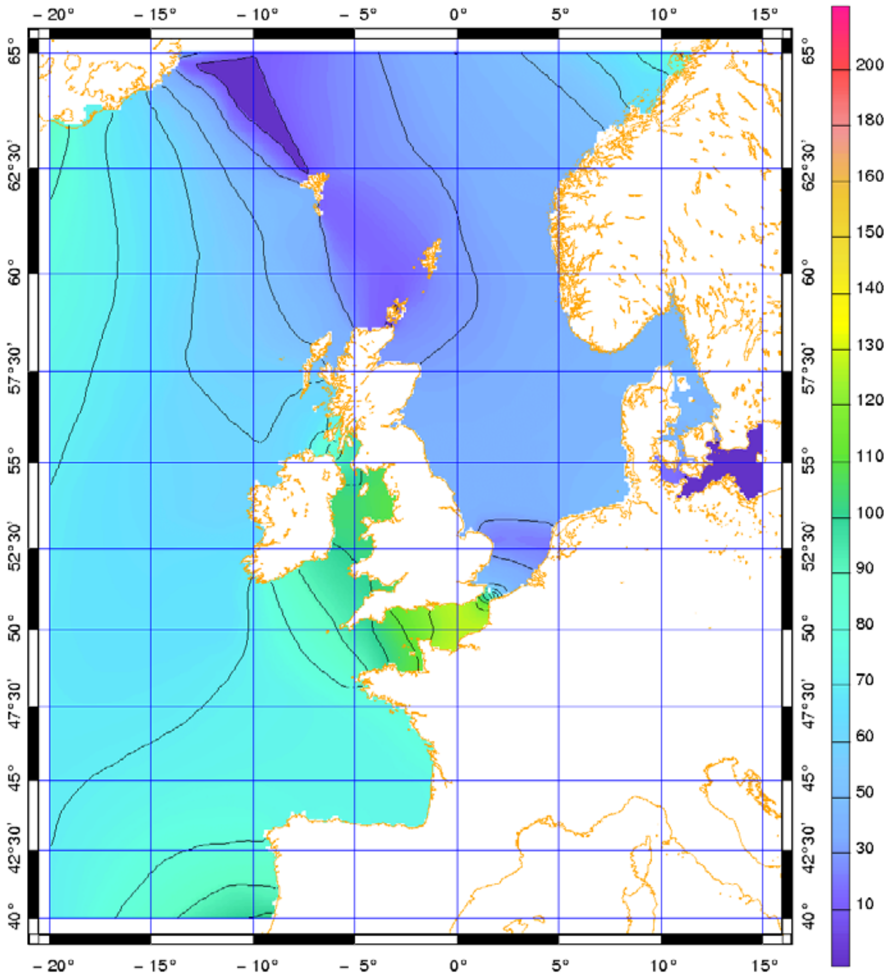
- Pressure gradient #1 ill-constrains tangential gradient (ill-defined Corilolis term)
- Pressure gradient #2 provides a full gradient, but is over-lapping

Pressure gradient scheme #1
Identical to C-grid

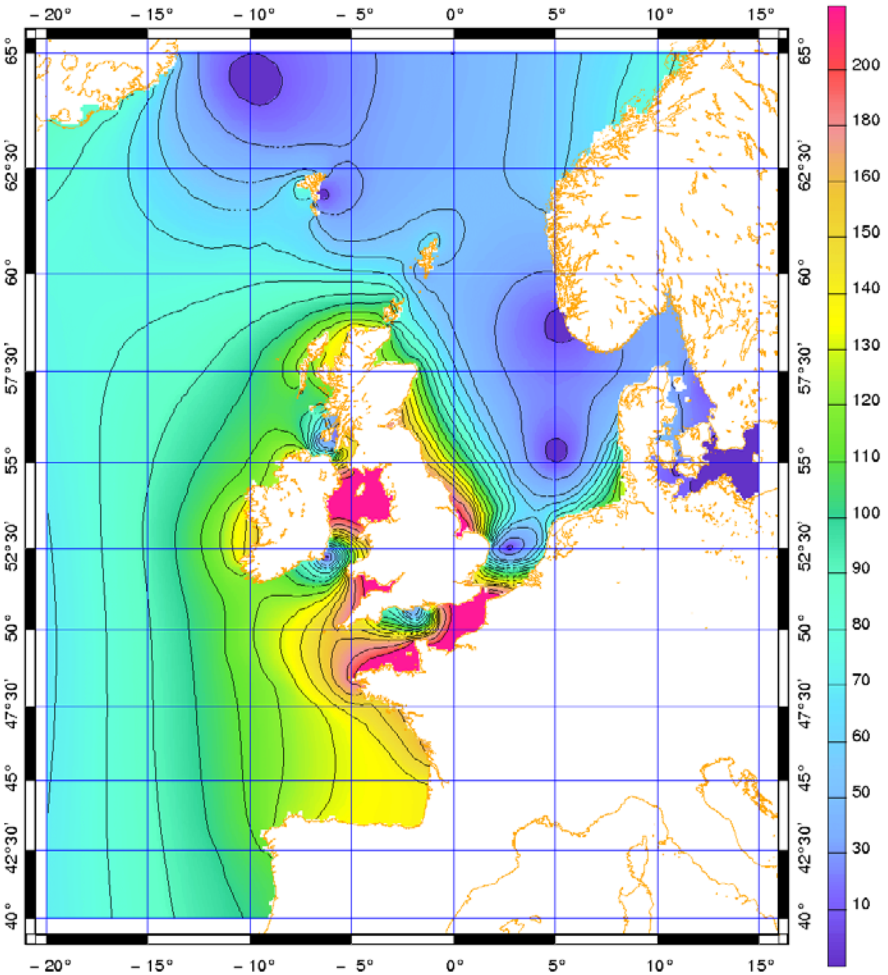
Pressure gradient scheme #2



Pressure gradient issue : impact on M2 tide simulation



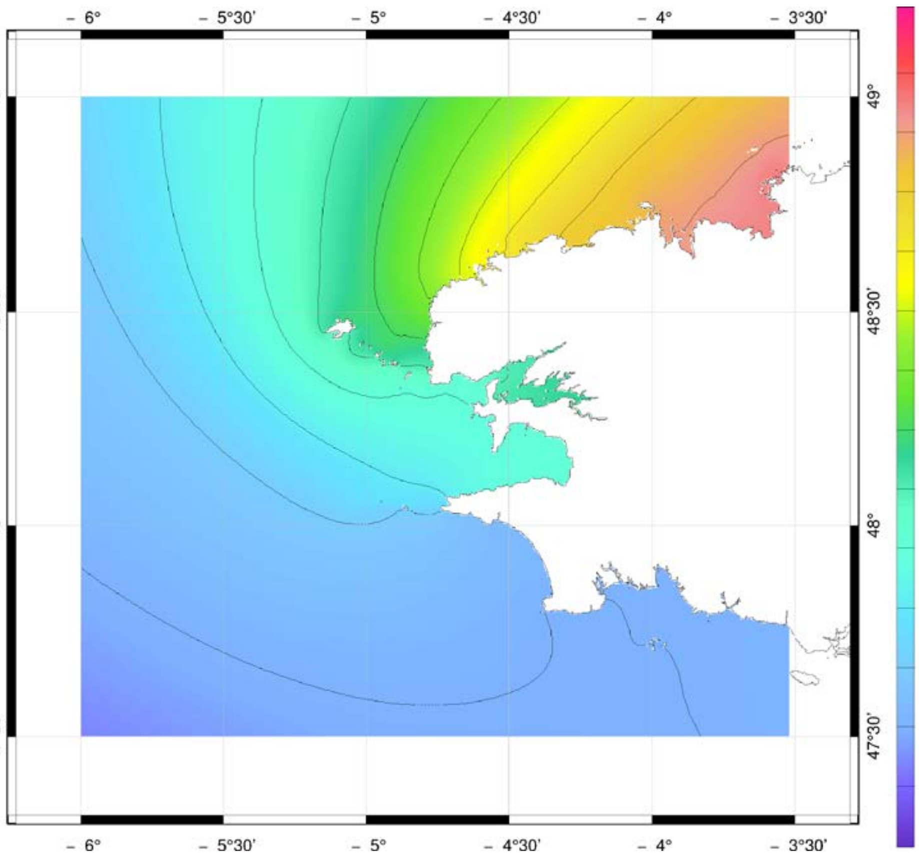
M2 elevation amplitude,
pressure gradient scheme #1
IFREMER MARS grid



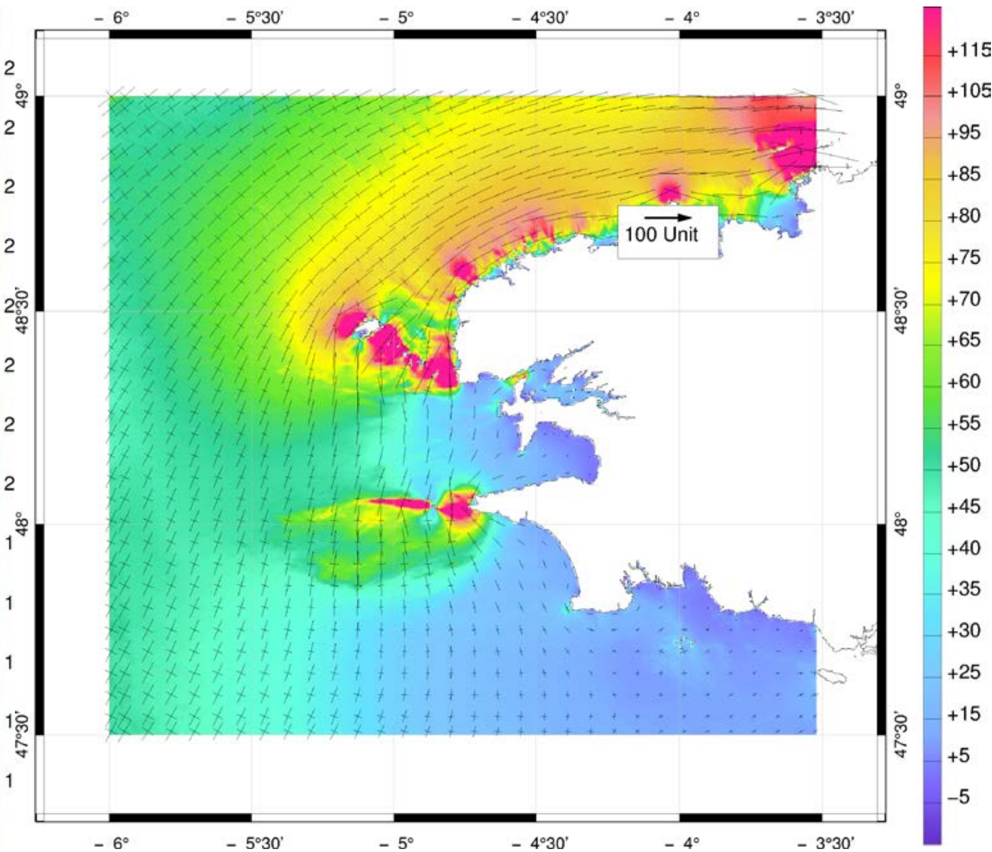
M2 elevation amplitude,
pressure gradient scheme #2
IFREMER MARS grid

Coriolis appropriate handling is essential…

HYCOM C-grid : Ushant Sea configuration



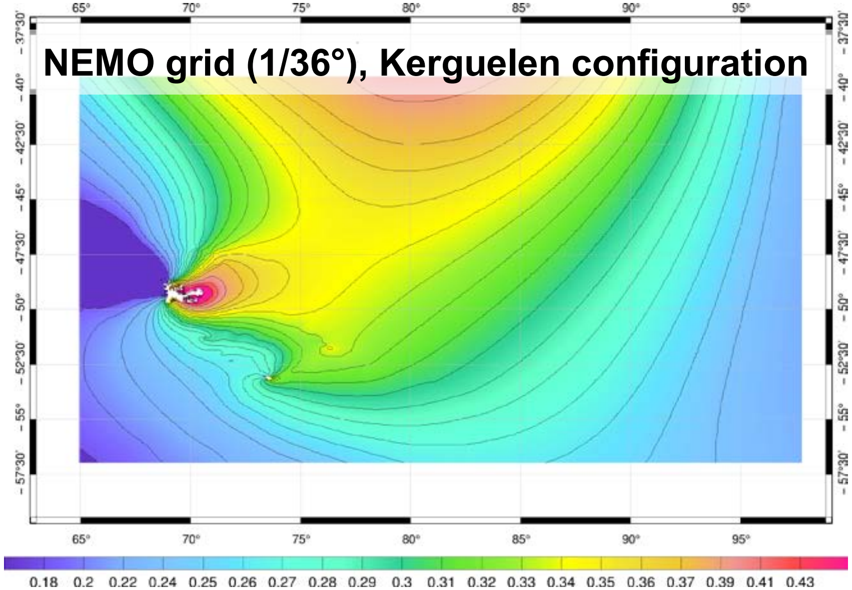
M2 elevation amplitude

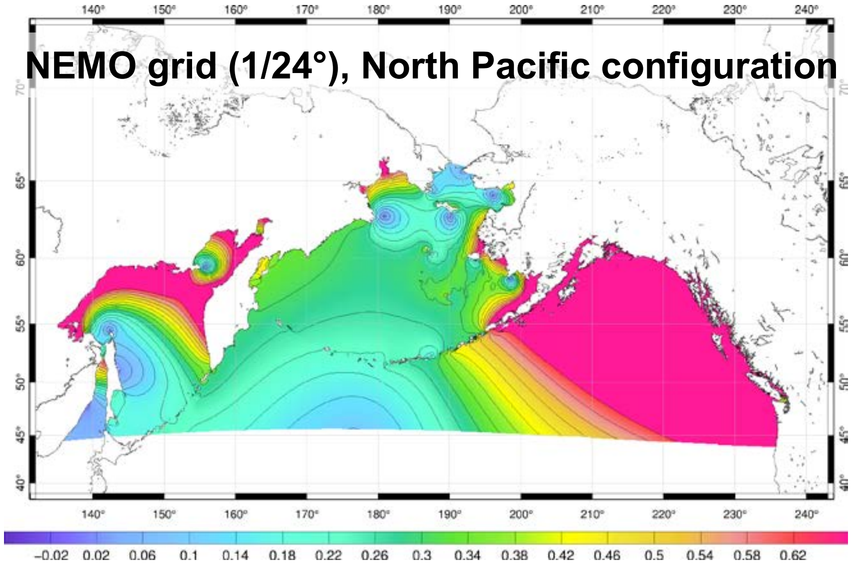


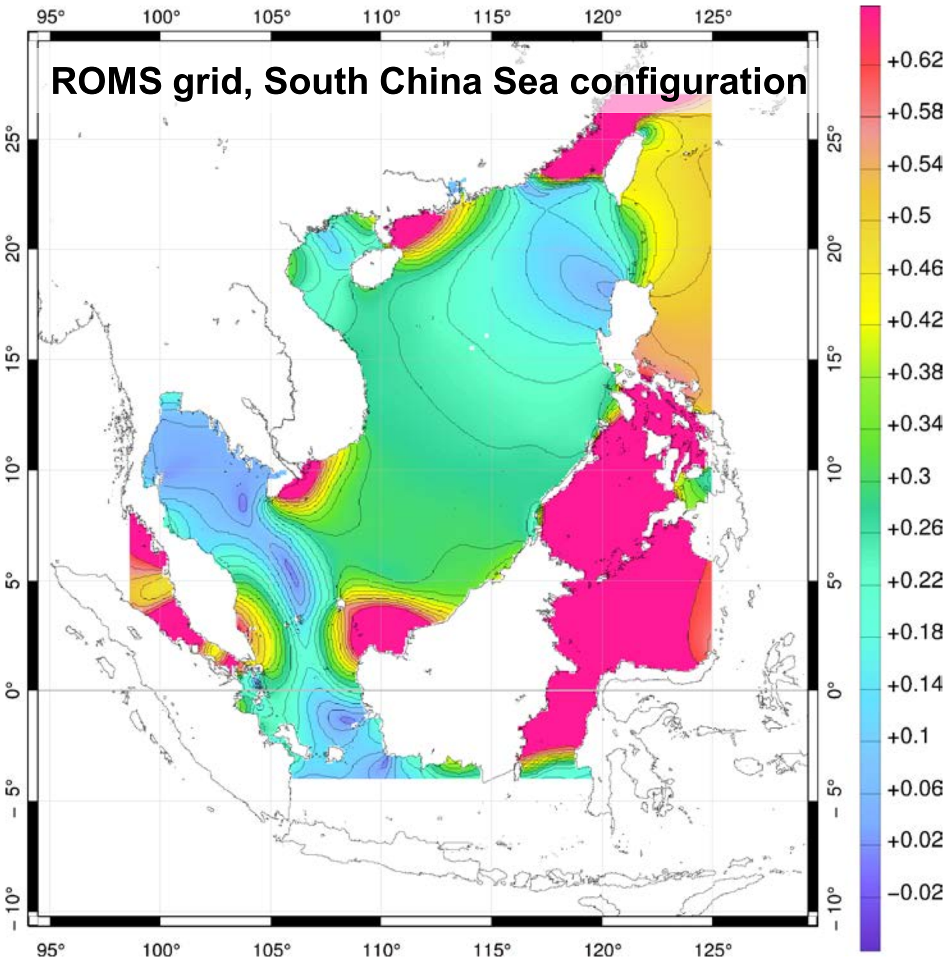
M2 tidal ellipses

Required structured model inputs: T-grid, bathymetry, T-landmask

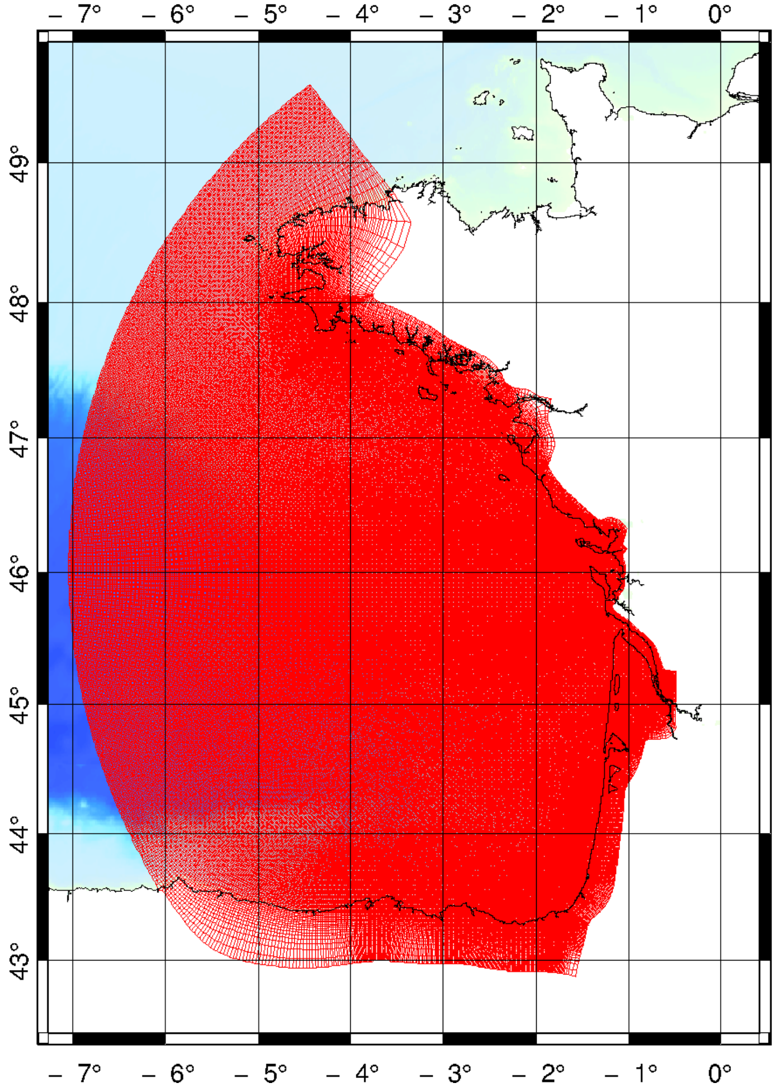
Some other examples



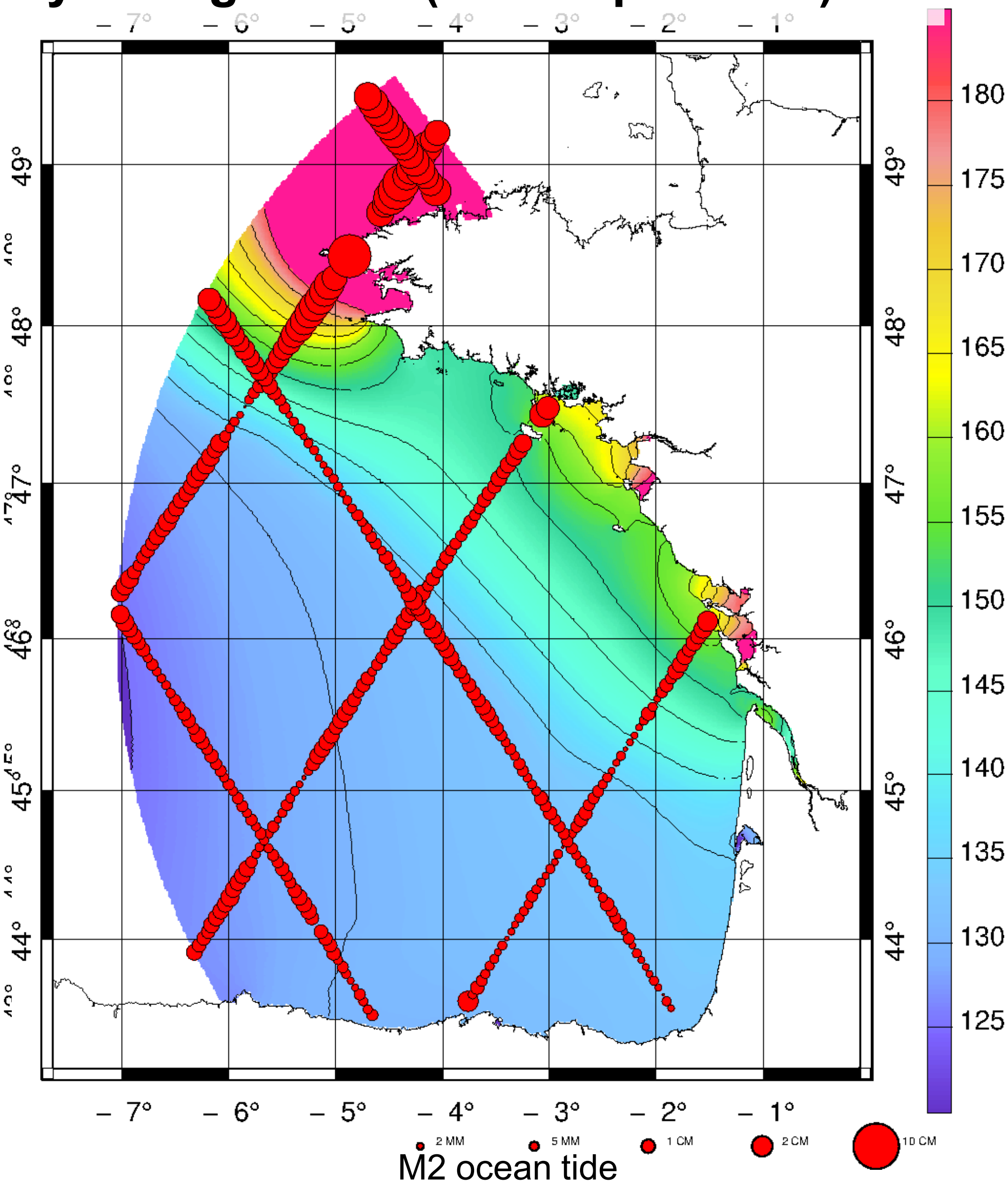




Symphonie grid: Bay of Biscay configuration (200m upto 3 km)



Finite difference grid



M2 ocean tide

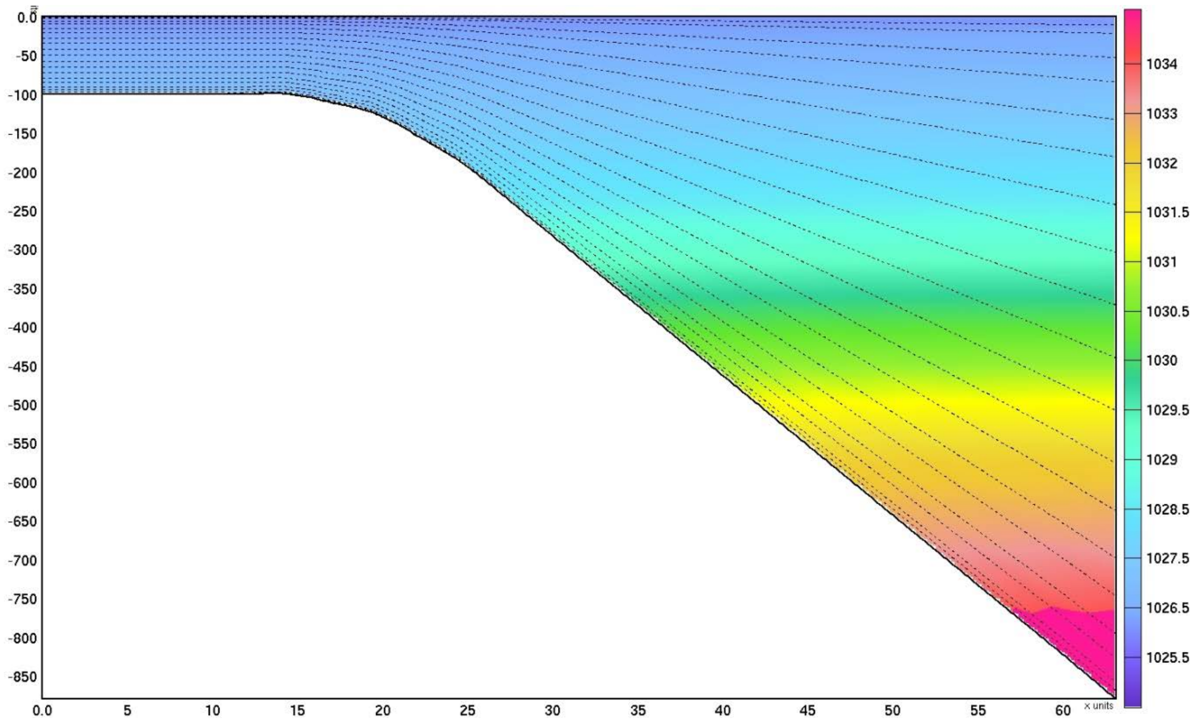
- T-UGOm spectral solver works on multi-element, multi-discretisation, structured and unstructured grids
- Efficient to produce tidal solutions, from global to estuarine configuration
- Useful to downscale global ocean tidal atlases onto regional configuration (consitent currents at open boundaries)

Triangle elements

3D spectral modeling

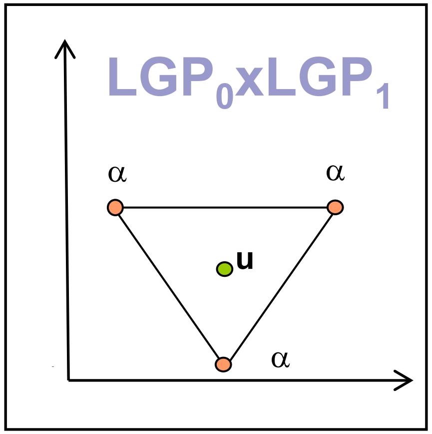
Vertical discretisation

- Layer model
- Similar to generalized sigma-level discretisation
- Density is varying horizontally, but uniform along the vertical inside the 3D element



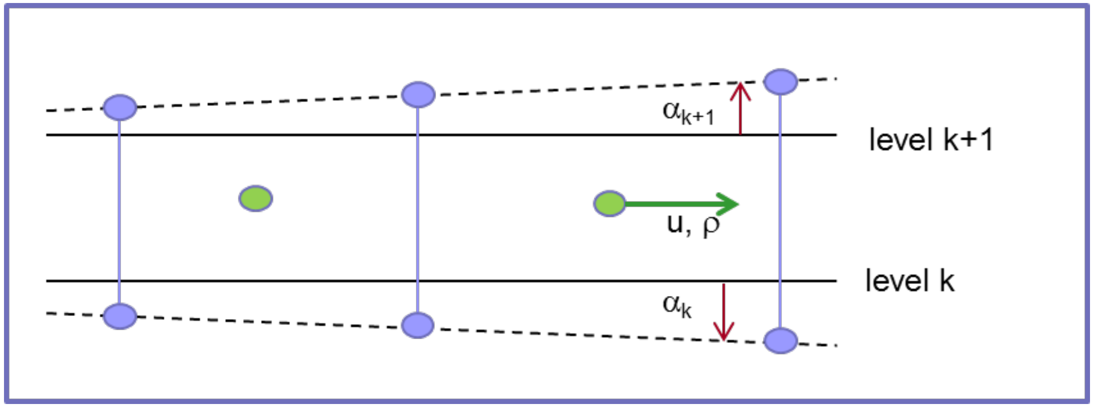
LGP0xLGP1, prismatic elements

- continuous, linear level displacements
- discontinuous, uniform velocities



Spectral 3D equations, basis

- Navier-Stokes equations vertically integrated inside layers (similar to generalized sigma-level equations)
- Impermeable layers (no vertical fluxes)
- No horizontal diffusion, horizontal advection in sub-harmonic forcing
- Unknowns are 3D currents and level displacements



Spectral 3D equations

Frequency-domain equations

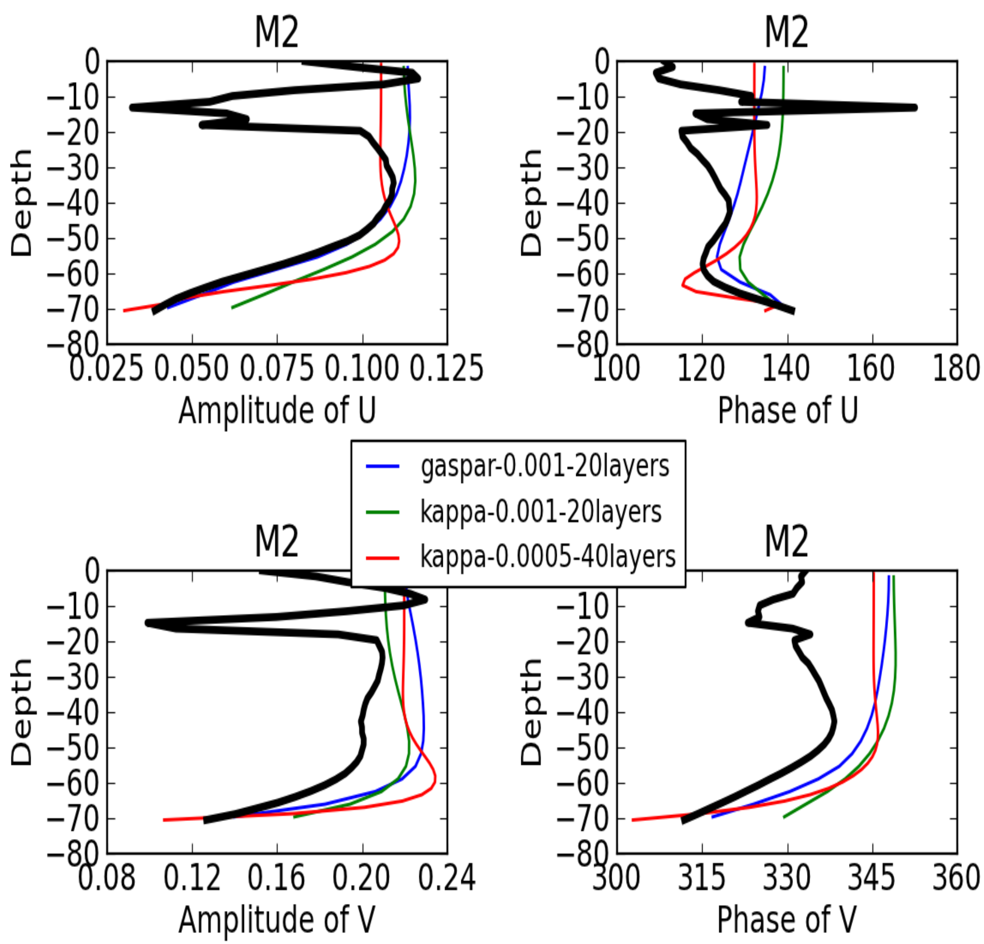
1 : level impermeability condition	$w = \varpi + \mathbf{v}\cdot\nabla_H s + \frac{\partial s}{\partial t} = \mathbf{v}\cdot\nabla_H s + \frac{\partial s}{\partial t}$
2: layer-integrated mass conservation +1	$j\omega\Delta\alpha + \nabla\cdot\overline{\Delta s}\mathbf{v} = j\omega\Delta\alpha + \nabla\cdot\mathbf{V} = 0$
3: layer-integrated momentum equation	$j\omega\mathbf{V} + f\times\mathbf{V} = -\frac{1}{\rho}\int_{s_0}^{s_1}\nabla_H p + g\overline{\Delta s}\nabla(\Pi-\delta) + \left[\kappa_v\frac{\partial\mathbf{v}}{\partial z}\right]_{s_0}^{s_1}$
4: bottom friction (vertical diffusion BC)	$\kappa_v\frac{\partial\mathbf{v}}{\partial z}_{bottom} = -C_D\lVert\mathbf{v}\rVert\mathbf{v}_{bottom}$

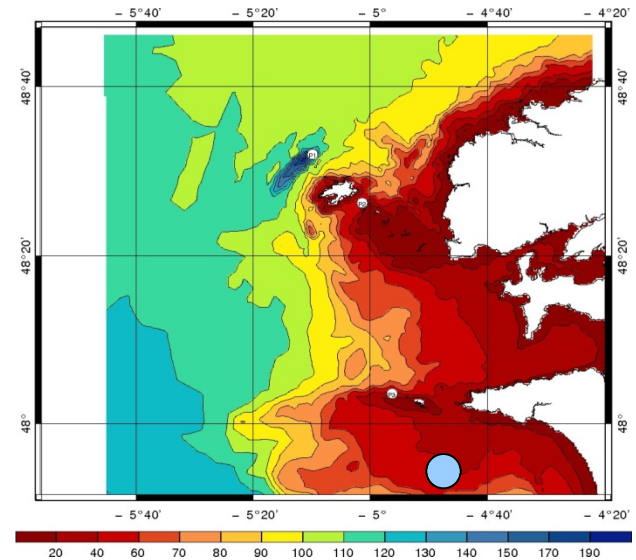
$\mathbf{u}$: 3D velocity
$\mathbf{v}$: horizontal velocity (2D)
$\mathbf{V}$: horizontal transport
w : vertical velocity
h : mean depth
s : layer interface position (immersion)
α : layer interface (level) displacement
Δs : layer thickness
$\overline{\Delta s}$: mean layer thickness
$\Delta\alpha$: layer thickness change
ϖ : omega velocity
ω : tidal pulsation

Wave equation

- Wave equation formed by substituting Eq. 3 in Eq. 2
- Turbulence (and vertical diffusion) and non-linear terms solved by iterations

3D barotropic turbulent scheme experiment

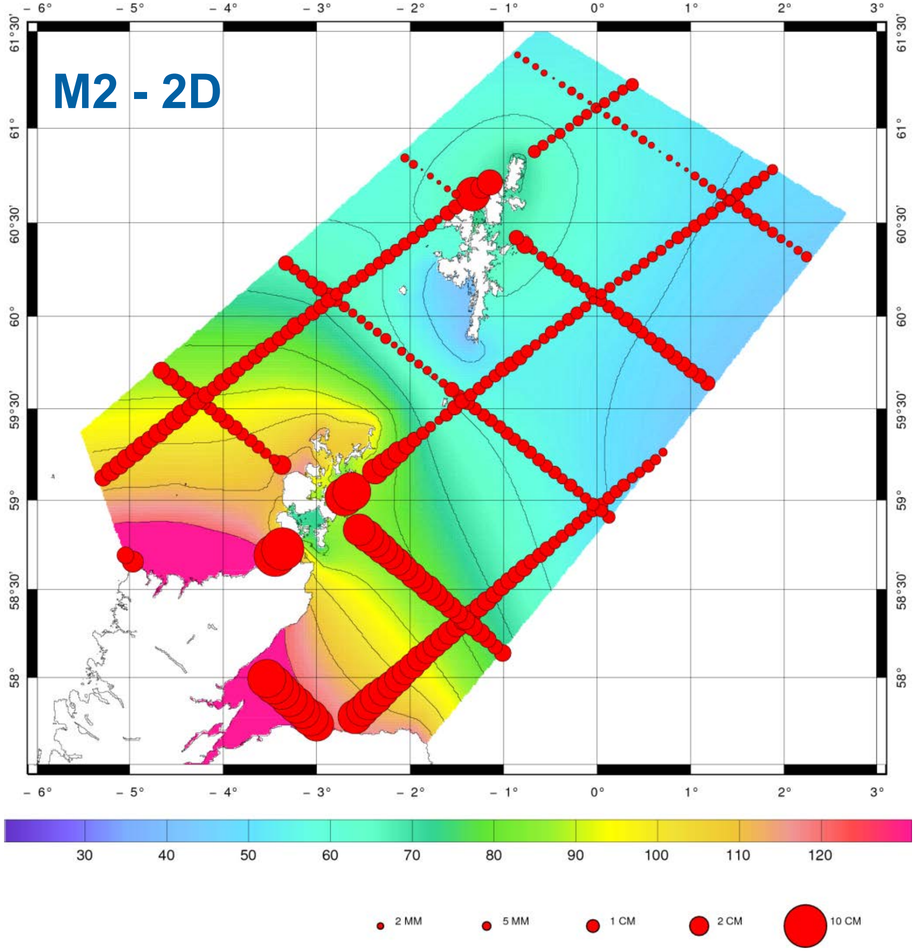


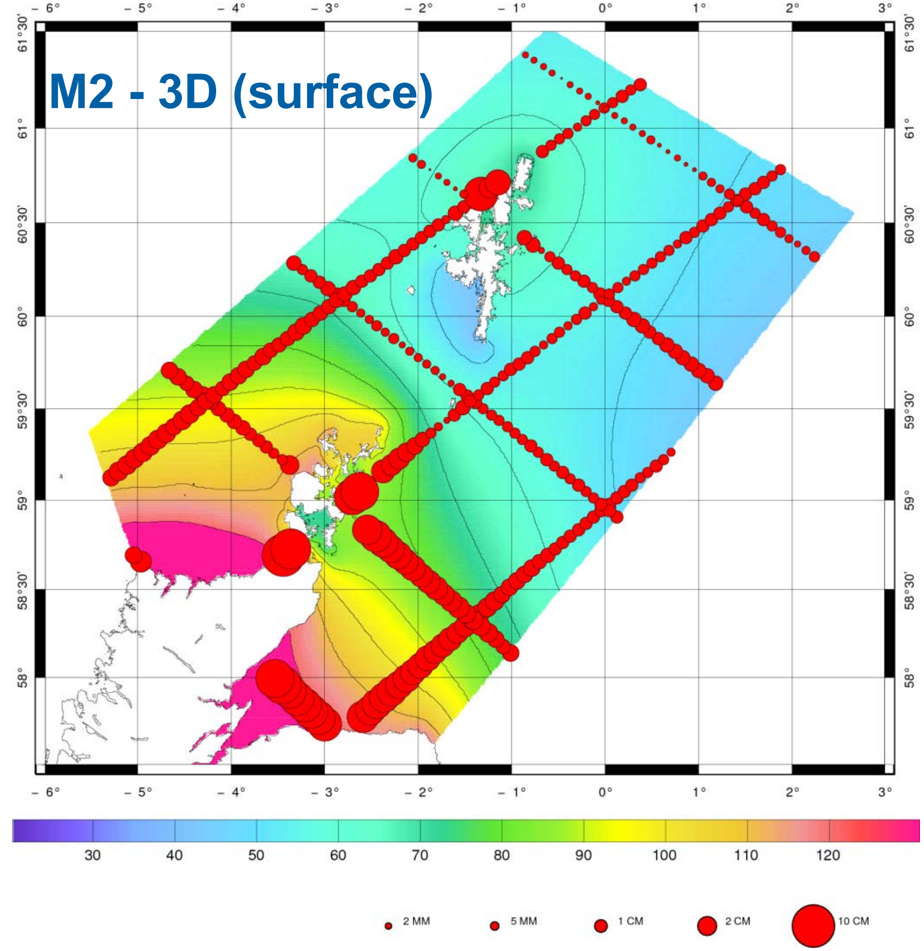


. Low resolution with high Kv can perform better than high resolution.

. The addition of a turbulent scheme (non constant Kv) can be highly beneficial in some cases.

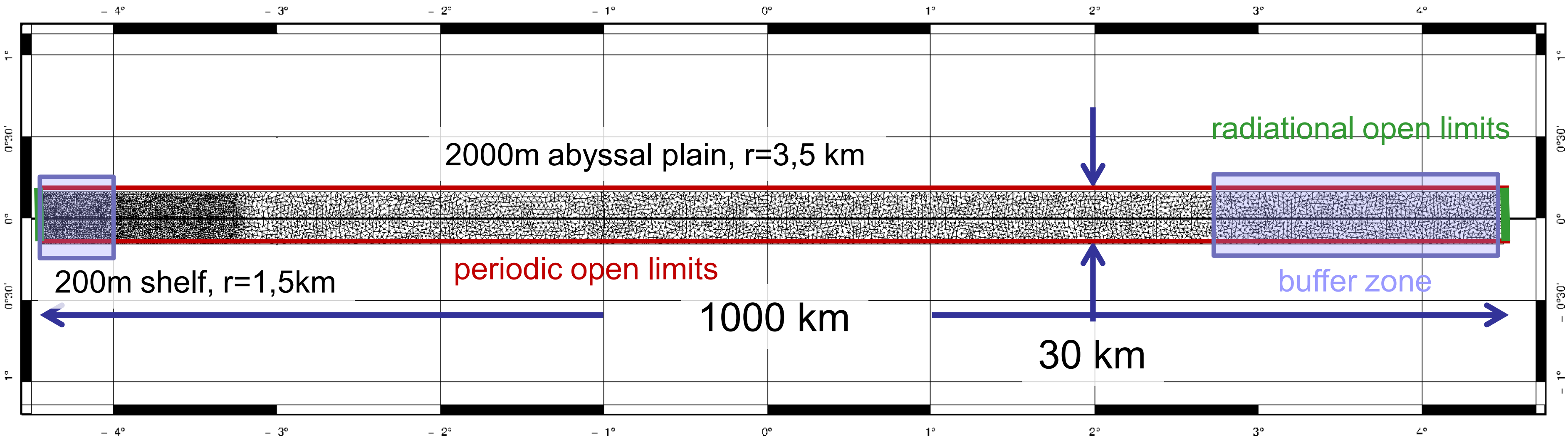
3D barotropic experiment

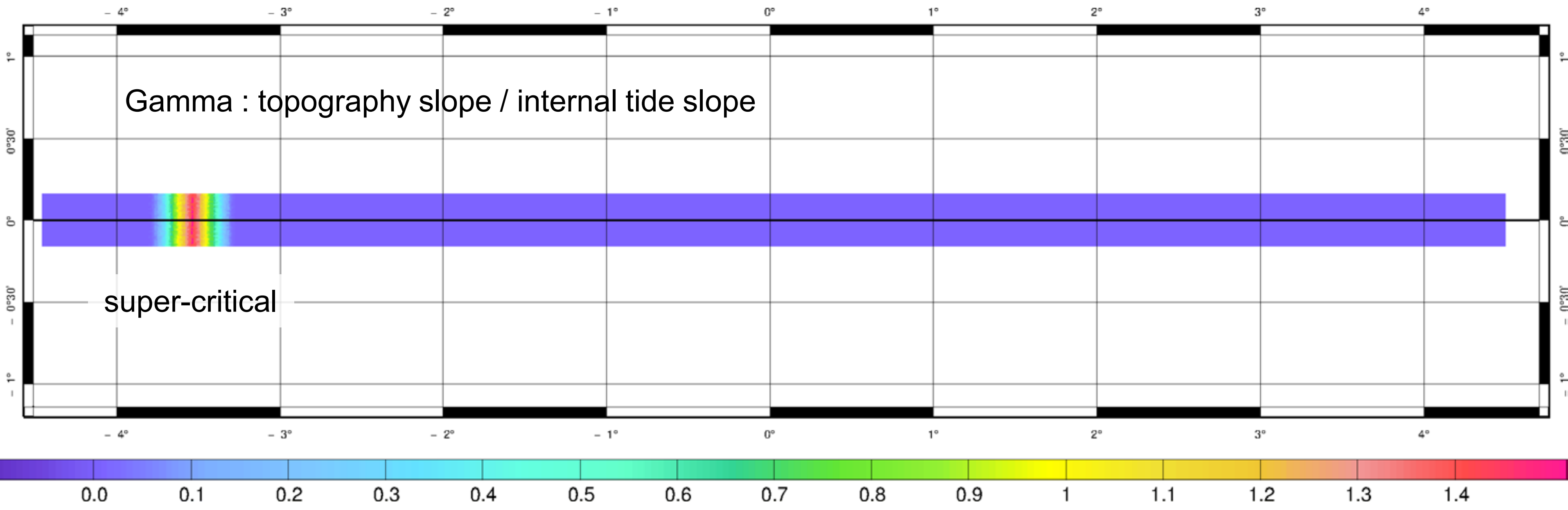




Vector difference (cm)	M2 - 2D	M2 - 3D (surface)	M2 - FES2012 (2D)
Tide gauges	3.6	3.4	3.6
Altimetry	2.1	2.0	2.5

Periodic channel test case: constant N=3.e-03, f=0





Spectral 3D baroclinic boundary conditions

- Perfectly Matched Layers (PML):
  - Absorbing buffer zone (reflectionless under certain conditions)
  - Tricky to implement (barotropic forcing issue), in progress
- Sommerfeld radiative condition:
  - Effective for mono-chromatic waves
  - Choice : address 1st baroclinic mode
  - In consequence, vertical diffusivity is smoothly but strongly increased in a boundary buffer zone to minimize higher modes

$$\frac{\partial \eta'}{\partial t} + c\nabla_H \eta' \cdot \mathbf{n} = -j\omega\eta' + c\nabla_H \eta' \cdot \mathbf{n} = 0$$

radiative condition (at each level), variational solver

$\mathbf{n}$: outward normal vector

s : layer interface position (immersion)

h : mean depth

η : level (layer interface) displacement

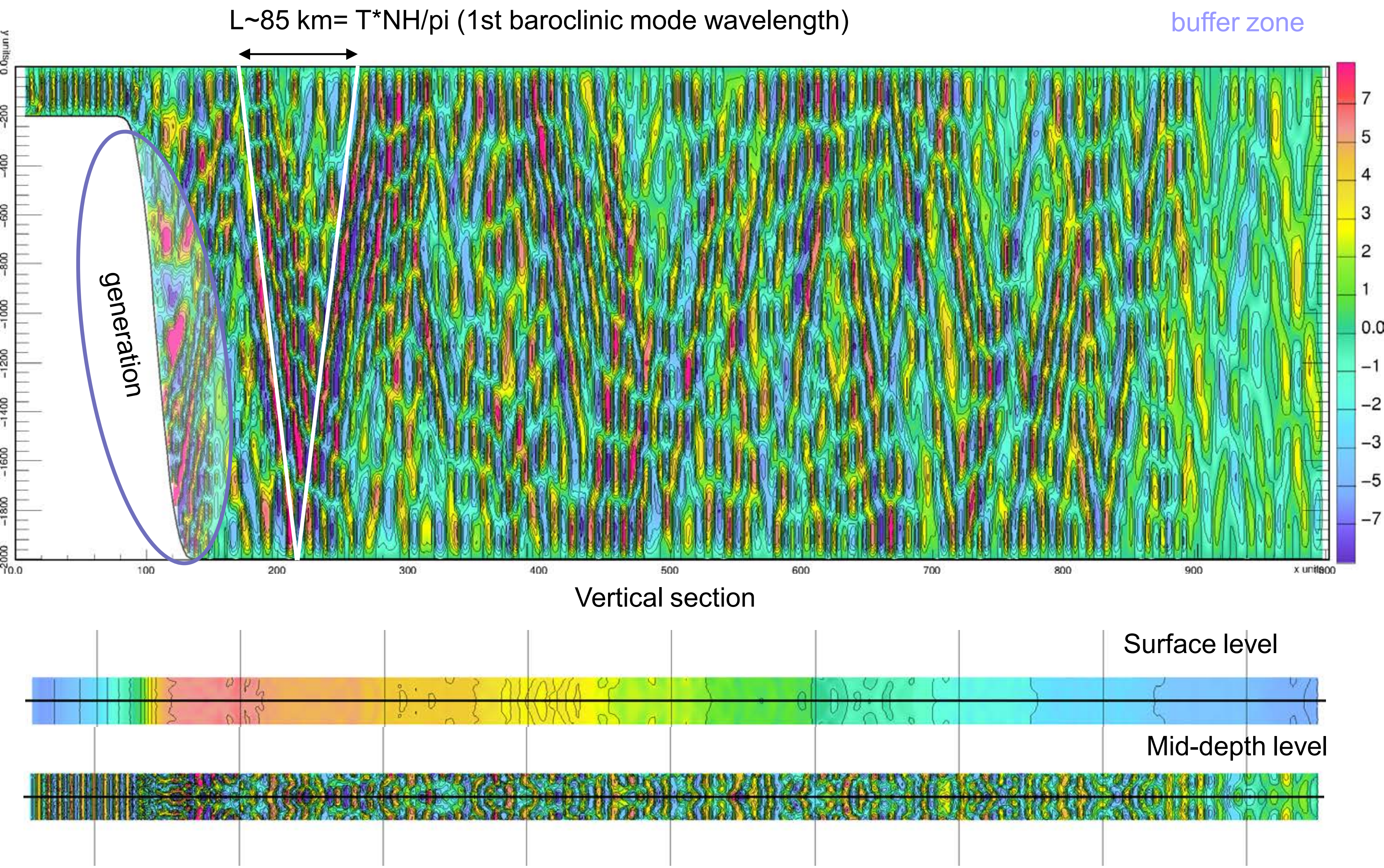
$\eta_b = \frac{s+h}{h}\eta_{surface}$: barotropic level displacement

$\eta' = \eta - \eta_b$: baroclinic level displacement

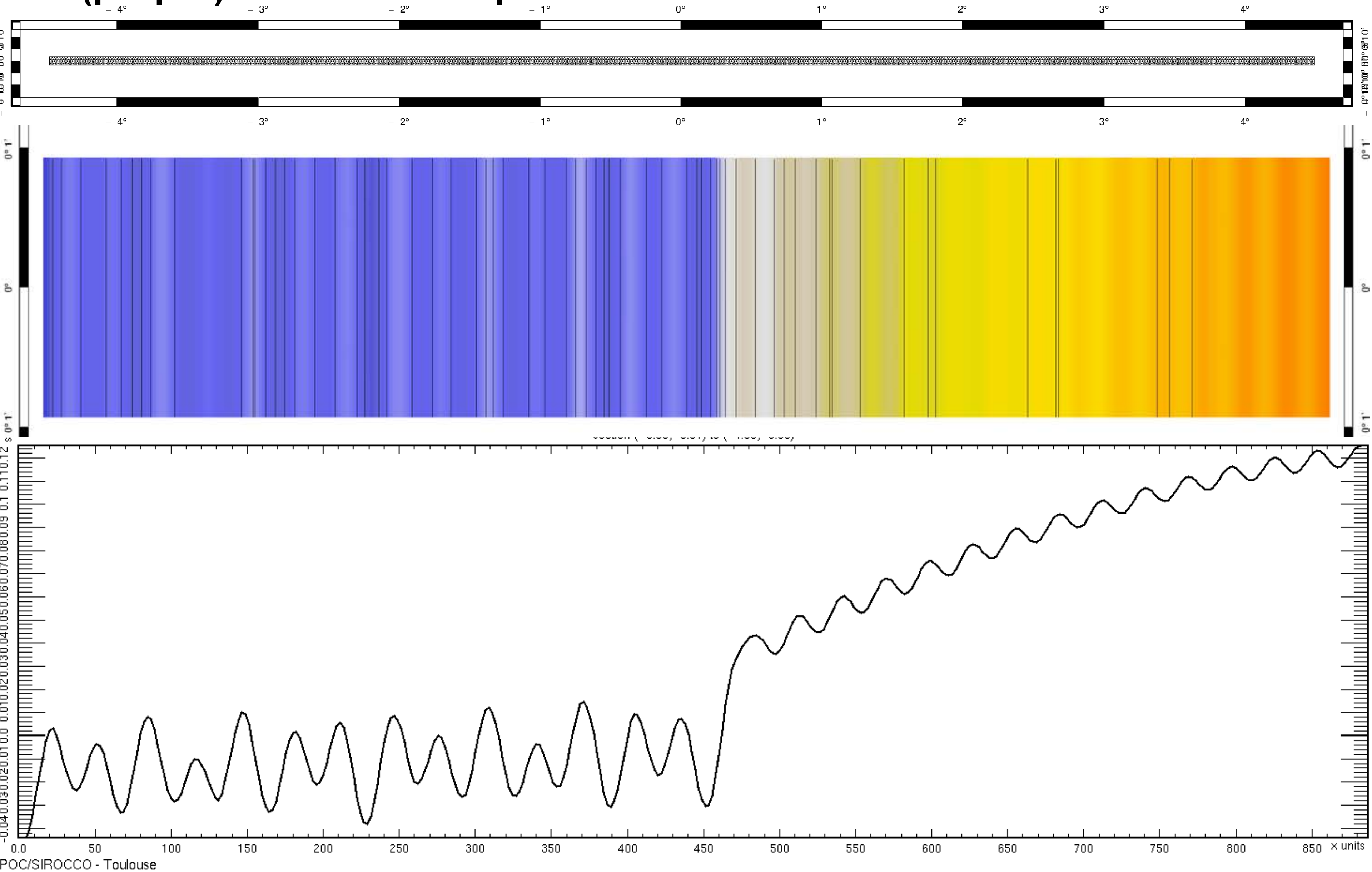
c : 1st baroclinic mode celerity

ω : tidal pulsation

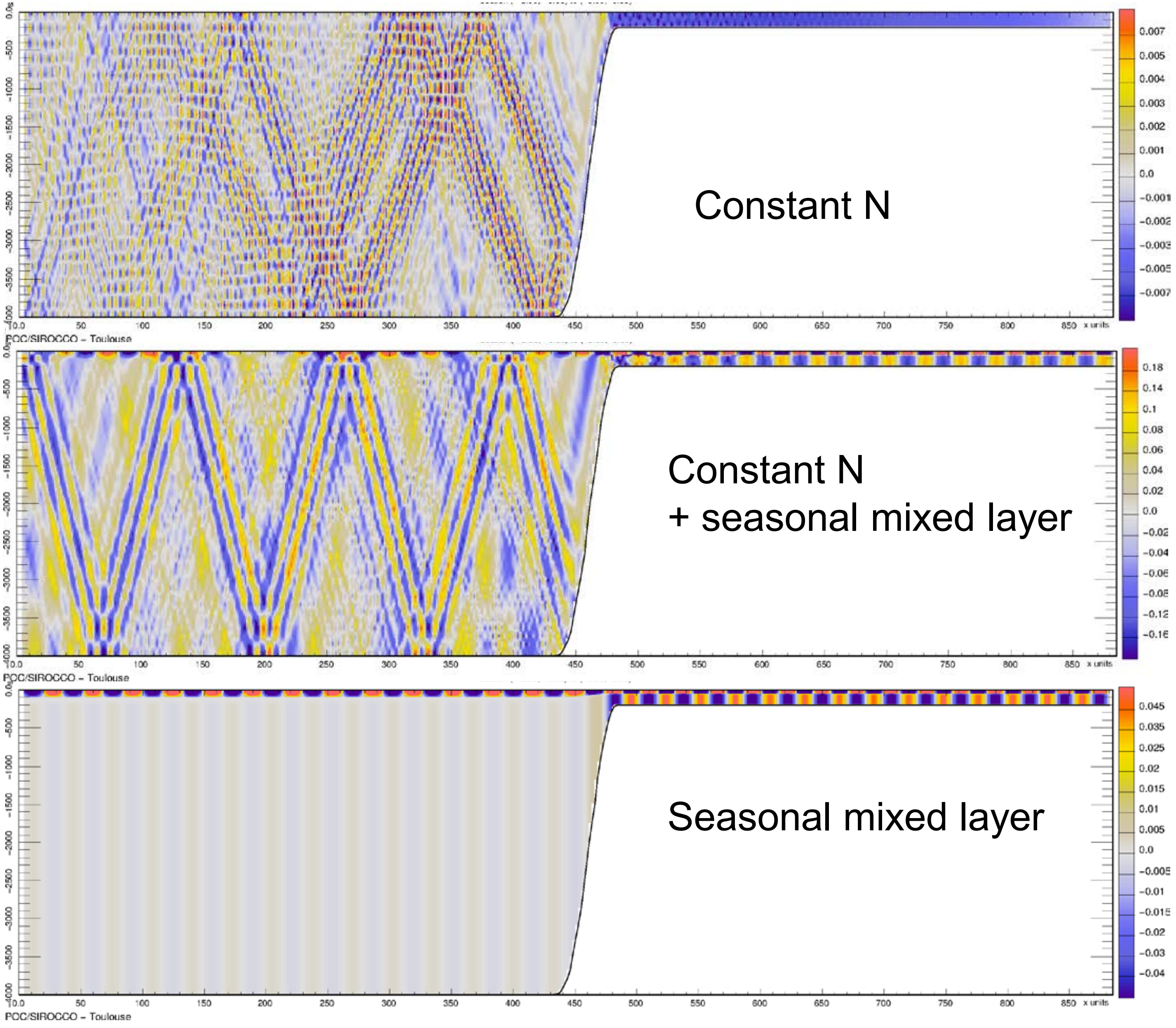
Instantaneous level displacement (t=0)



(proper) Plane wave experiment



(proper) Plane wave experiment



Analytical solutions (2D and separated-modes 3D)

Tidal linearised, flat-bottom equations

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x} - ru$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y} - rv$$

$$\frac{\partial \eta}{\partial t} = -H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}\right)$$

Tidal frequency-domain ("spectral") equations

$$(-i\omega + r)\mathbf{u} + 2\vec{\Omega} \times \mathbf{u} = -g\nabla\eta$$

$$\frac{\partial \eta}{\partial t} + H\nabla\cdot\mathbf{u} = -i\omega\eta + H\nabla\cdot\mathbf{u} = 0$$

$$\mathbf{\Omega}' = \begin{bmatrix} 0 \\ 0 \\ \Omega\sin\varphi \end{bmatrix} \qquad 2\mathbf{\Omega}'\times\mathbf{u} = 2\Omega\begin{bmatrix} -v\sin\varphi \\ u\sin\varphi \\ 0 \end{bmatrix}$$

$$(-i\omega + r)^2 + f^2 = \delta$$

$$c^2 = gH$$

Dispersion relation:

$$-\frac{c^2}{\delta}\left[(-i\omega + r)\nabla\cdot\nabla\eta + \frac{2f}{\delta}\nabla f\cdot\left((-i\omega + r)\nabla\eta + 2\vec{\Omega}'\times\nabla\eta\right)\right] = i\omega\eta$$

Let's try pretty simple

- *f* taken as constant
- no friction

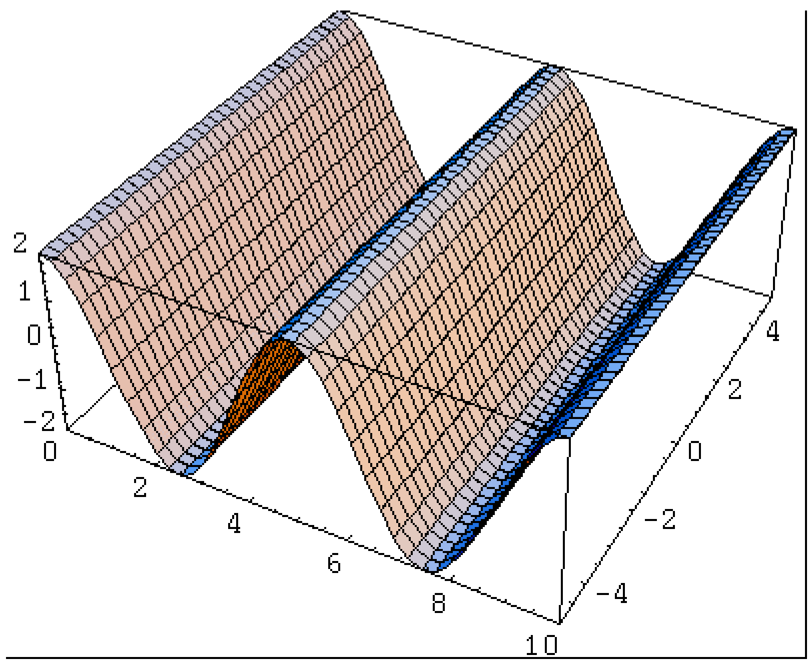
Dispersion relation reduces to:

$$\frac{c^2}{f^2-\omega^2}\nabla\cdot\nabla\eta=\eta$$

Plane waves: $\eta(x,y,t)=\eta e^{i(k_x x+k_y y-\omega t)}$ ⟶ $\nabla\cdot\nabla\eta=-\left(k_x{}^2+k_y{}^2\right)\eta=-k_H{}^2\eta$

$$k_H=\frac{1}{c}\sqrt{\omega^2-f^2}\qquad \lambda=\frac{2\pi}{k_H}=\frac{\omega}{\sqrt{\omega^2-f^2}}cT$$

- ➢ Wavelength increase with latitude, inifinite at critical latitude
- ➢ No solution above critical latitude
- ➢ 1D wavelength at low latitudes $k_H=\frac{\omega}{c}\quad \lambda=cT=\sqrt{gH}T$
- ➢ Not suitable to describe notable structures such as amphidromic points



Let's try harder

- *f* taken as constant
- no friction

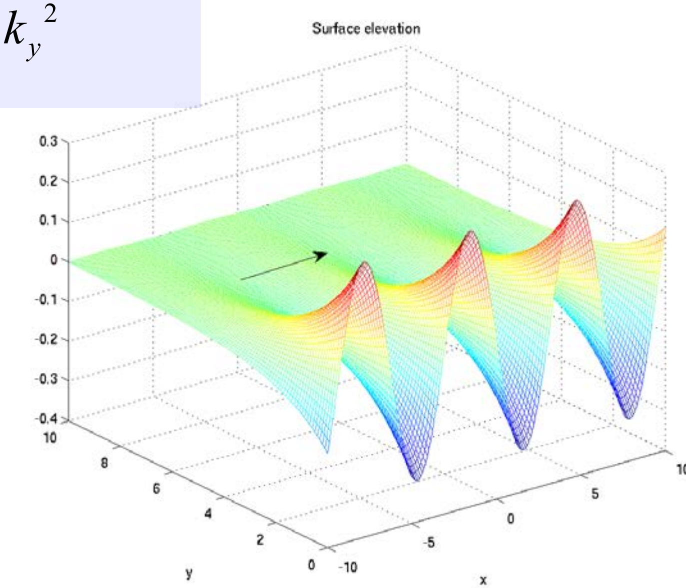
Dispersion relation reduces to:

$$\frac{c^2}{f^2-\omega^2}\nabla\cdot\nabla\eta=\eta$$

Kelvin waves: $\eta(x,y,t)=\eta e^{-k_y y+i(k_x x-\omega t)} \longrightarrow \nabla\cdot\nabla\eta=-\left(k_x^{\,2}-k_y^{\,2}\right)\eta$

$$k_H=k_x=\frac{1}{c}\sqrt{\omega^2-f^2+k_y^{\,2}} \qquad \lambda=\frac{2\pi}{k_H}=\frac{\omega}{\sqrt{\omega^2-f^2+k_y^{\,2}}}cT$$

- ➢ Coupling between across damping scale and wavelength
- ➢ critical latitude is no more an issue
- ➢ NOT 1D wavelength at low latitudes



Let's try even harder: amphidromic point

- ~~*f* taken as constant~~
- no friction

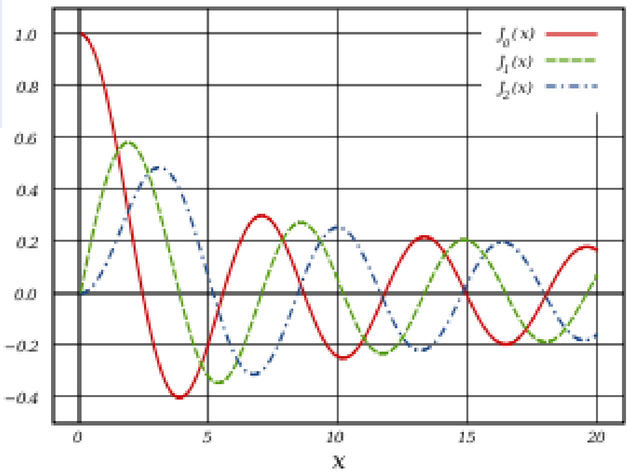
Dispersion relation reduces to:

$$\frac{c^2}{f^2-\omega^2}\nabla\cdot\nabla\eta=\eta$$

Axial solution: $\eta=\Phi(r)e^{i(\theta-\omega t)}$ ⟶ $\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\eta}{\partial\rho}\right)-\frac{1}{\rho^2}\eta=-\frac{(\omega^2-f^2)}{c^2}\eta=-\kappa^2\eta$

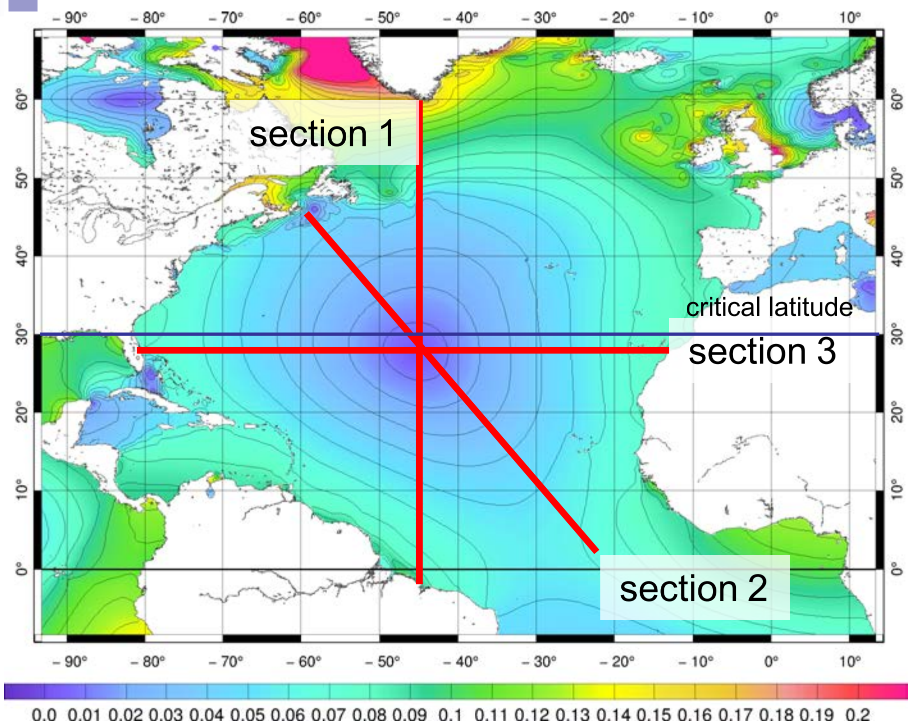
It is a Sturm-Liouville problem, more exactly a Bessel differential equation

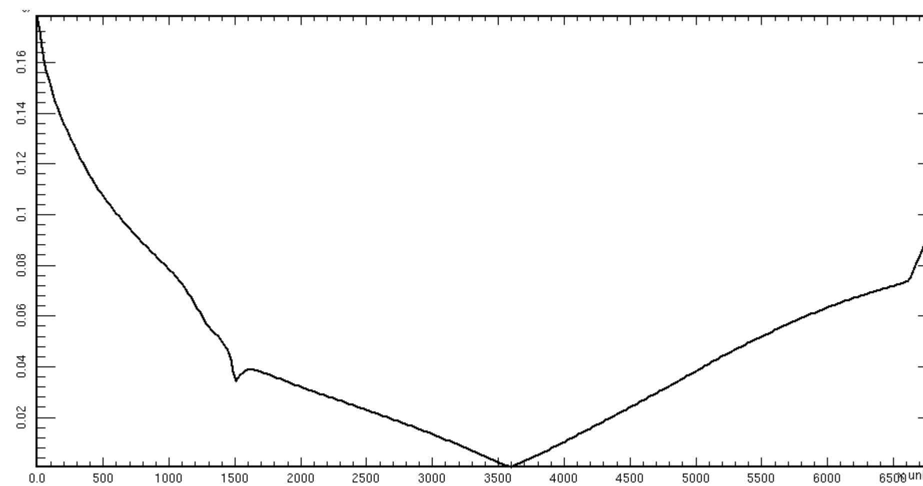
$$\eta=J_{\alpha=1}(\kappa\rho)=\sum_{m=0}^{+\infty}\frac{(-1)^m}{m!\,\Gamma(m+\alpha+1)}\left(\frac{x}{2}\right)^{2m+\alpha}=\sum_{m=0}^{+\infty}\frac{(-1)^m}{m!\,(m+2)!}e^{(2m+\alpha)\log\left(\frac{\kappa\rho}{2}\right)}$$



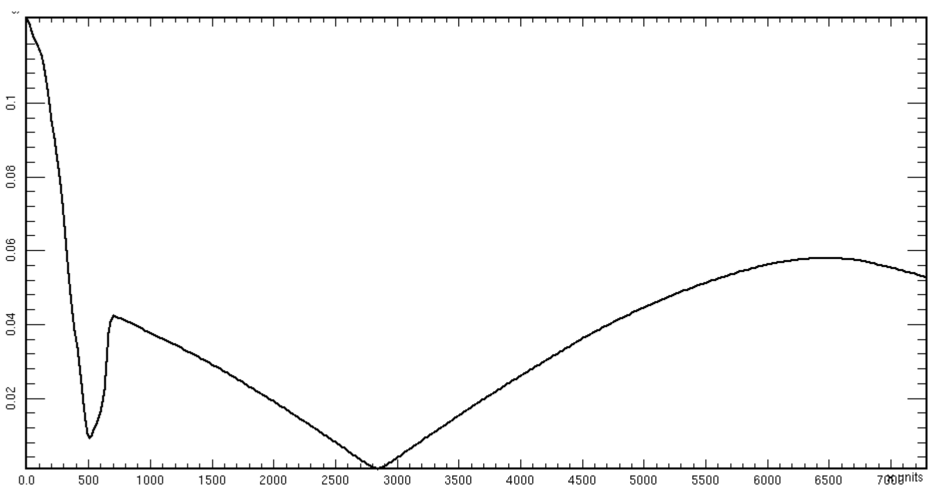
- ➢ Coupling between across scale and wavelength
- ➢ critical latitude solution $\eta=\eta_0\frac{\rho}{\rho_0}e^{i(k_\theta\theta-\omega t)}=\eta_0\frac{\rho}{\rho_0}e^{i(\theta-\omega t)}$

K1 ocean tide

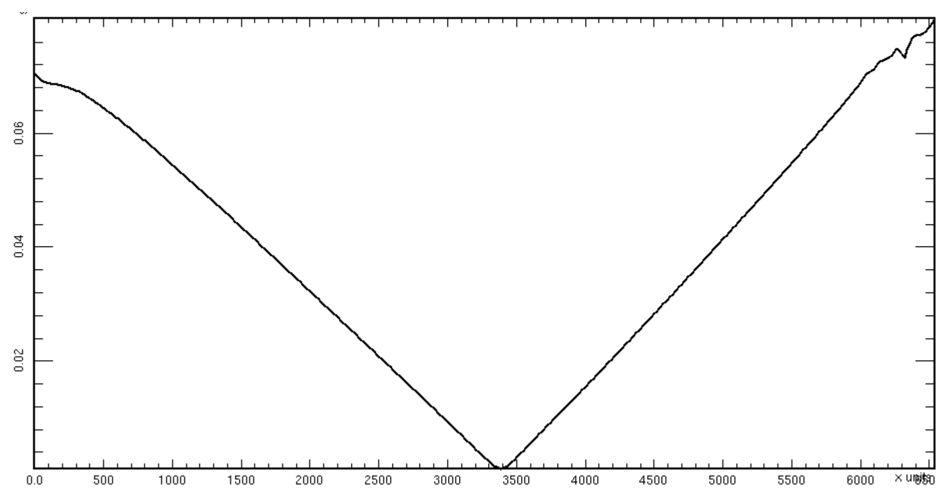




K1 amplitude profile, section 1



K1 amplitude profile, section 2



K1 amplitude profile, section 3

- How much do we know about internal tides:
  - Temporal varibility
  - Horizontal scales (dispersion relation)
  - Is horizontal-vertical mode separation approach robust enough
  - Are higher modes predictable
  - Dissipation processes

- More analytical work needed (i.e. not only plane waves)

- How/where can we setup realistic configurations for modelling
  - Open boundary condition issues
  - Validation

Summary

- Frequency-domain tidal dynamics solver is nested inside T-UGOm time-sequential solver
- Allows consistent downscaling for structured and unstructered models/configurations
- It solves for : 2D shallow-water, 2D + 1DV, 3D barotropic (shelf seas) and 3D baroclinic (internal tides)
- Accommodate different discretisation/element (generic coding)
- Frequency-domain data assimilation friendly
- Efficient for model parameters exploration
  - cheap
  - accurate
- On-going development (MPI) to address large domain 2D/3D simulations (10^{e+8} dof)

Thank you for attention