Using the Radon Transform to detect anisotropic error sources

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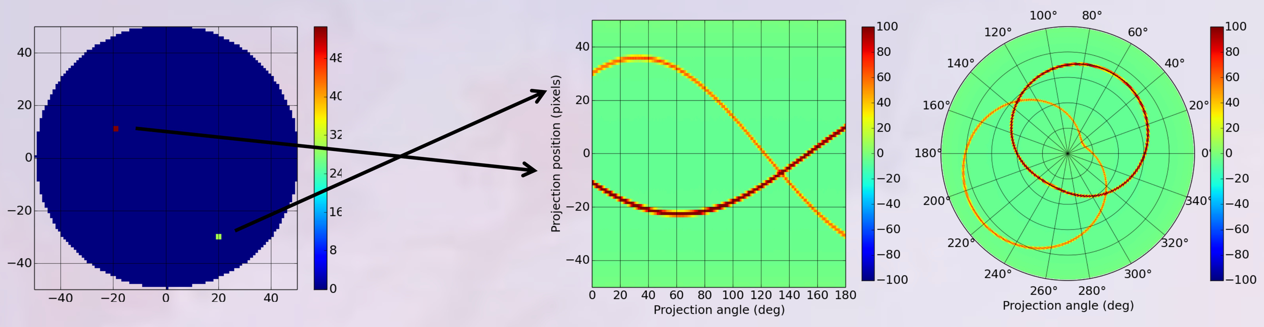
Radon Transform 101

The Radon transform (RT) is the integral transform of a density function over straight lines with angle α and offset distance s (F1).

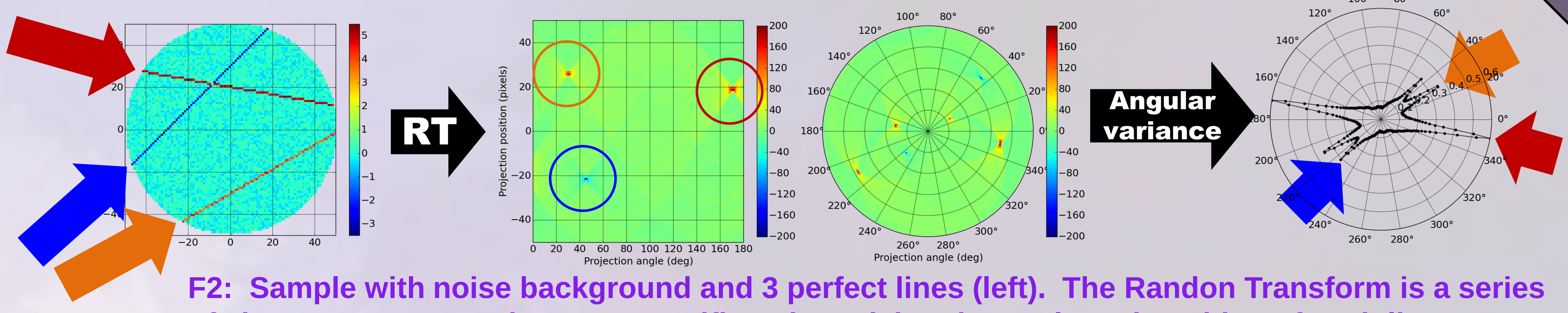
$$Rf(\alpha,s)=\int_{-\infty}^{\infty} f(x(t),y(t))\,dt = \int_{-\infty}^{\infty} f\big((t\sin\alpha+s\cos\alpha),(-t\cos\alpha+s\sin\alpha)\big)\,dt$$



F1: Sample with zero background and 2 non-zero pixels (left). Its Radon Transform is a sine wave for each pixel (middle), i.e. circle or cardioid curve in polar coordinates (right)

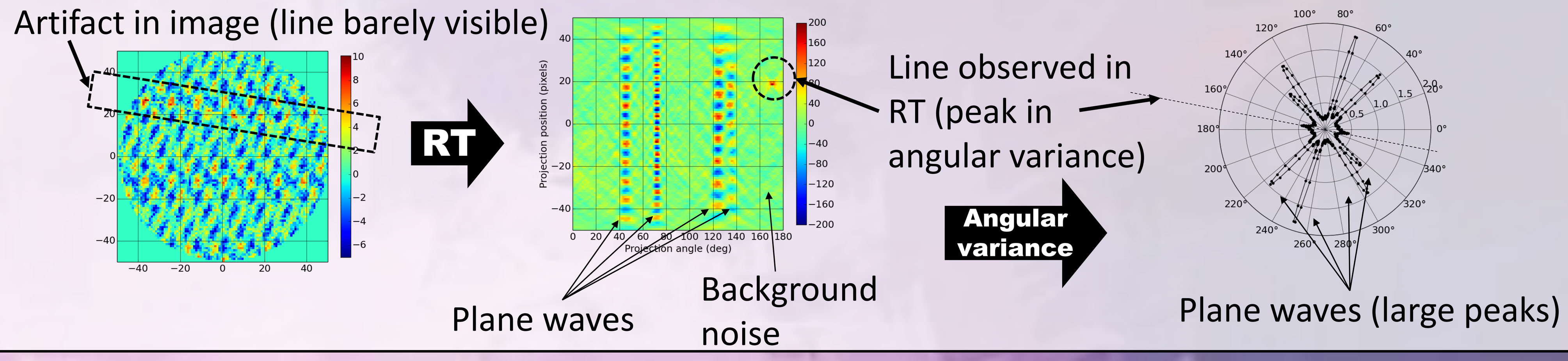
A major benefit of the RT is to detect alignments of coherent structures in a 2D or 3D field: plane wave, line, small set of aligned structures. Analyzing each sample is sometimes complex and the angular variance of the RT is also a very good metric (F2 and F3).

SIMPLE EXAMPLES



F2: Sample with noise background and 3 perfect lines (left). The Randon Transform is a series of sine waves converging on 3 specific points giving the angle and position of each line (middle left, or middle right). The angular variance of the RT yields a peak for each line (right).

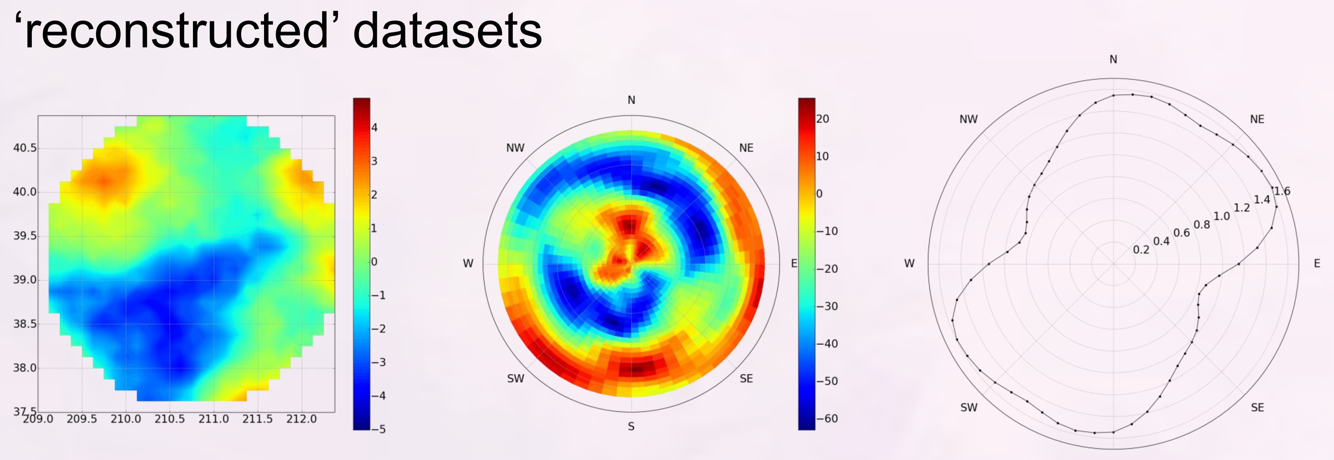
F3: Sample with noise background, four plane waves with interference and a barely visible isolated line (left). The Randon Transform shows a complex pattern for each component (middle left, or middle right). The angular variance of the RT can separate and quantify precisely each angular component (right).



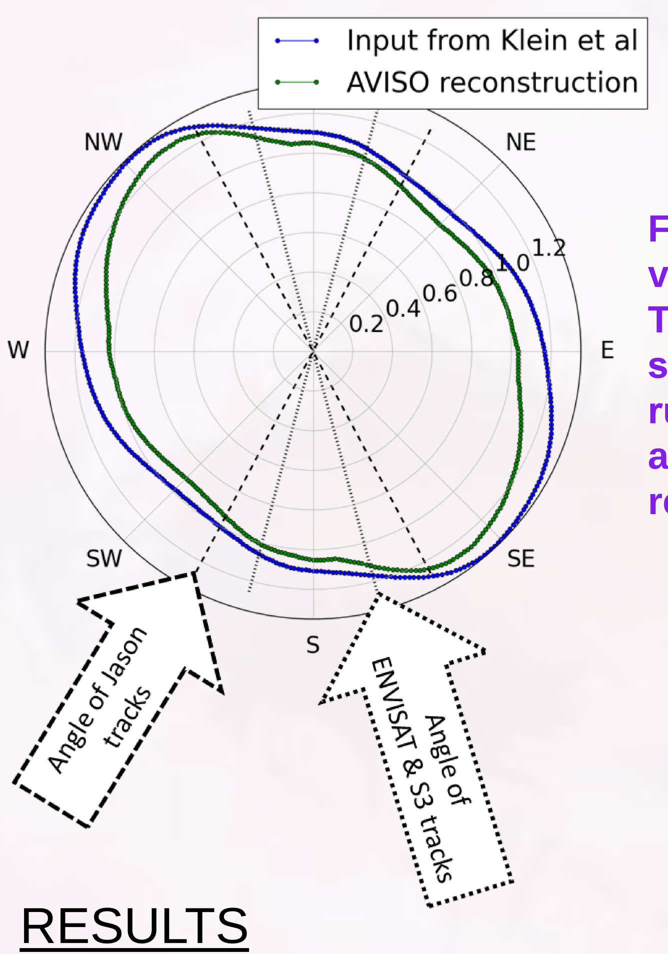
Use case #1: gauging the blind angle in AVISO maps

METHODOLOGY

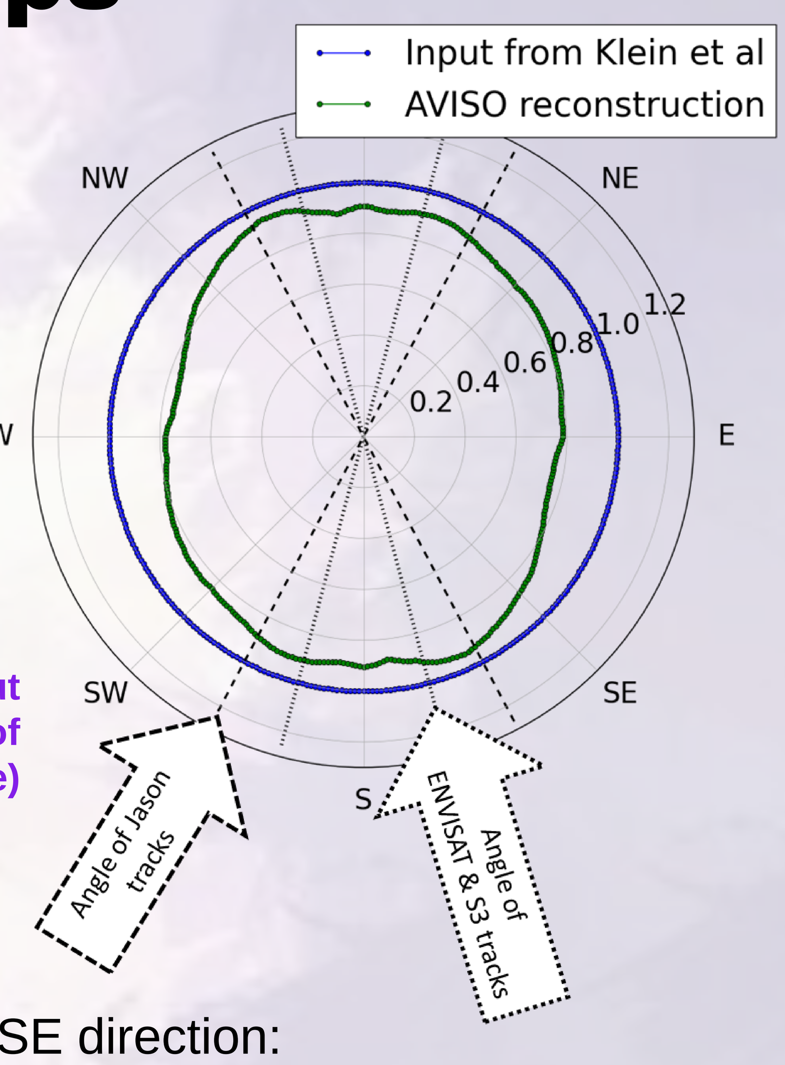
- OSSE approach with data from Pujol et al (2012)
- Take ocean model data as 'nature run' (Klein et al, 2008)
- Relocate it near 38°N (angle of satellite tracks)
- Sample the reality under satellite tracks
- Add realistic measurement errors
- Reconstruct 2D maps like AVISO/DUACS
- Take hundreds of random image pieces (~150 km radius)
- Compute local RT and angular variance for each piece (F4)
- Individual RT are largely dominated by the oceanic signal
- Compare the mean RT angular variance of 'nature run' and 'reconstructed' datasets



F4: Arbitrary snippet of AVISO map reconstruction (left), and its Radon Transform (middle) and angular variance of the RT (right)



F5: Mean angular variance of the Radon Transform for hundreds snippets from the 'nature run' of Klein et al (blue), and of the AVISO-like reconstruction (green)



F6: Same figure as F5 but normalized with RT of nature run (blue curve)

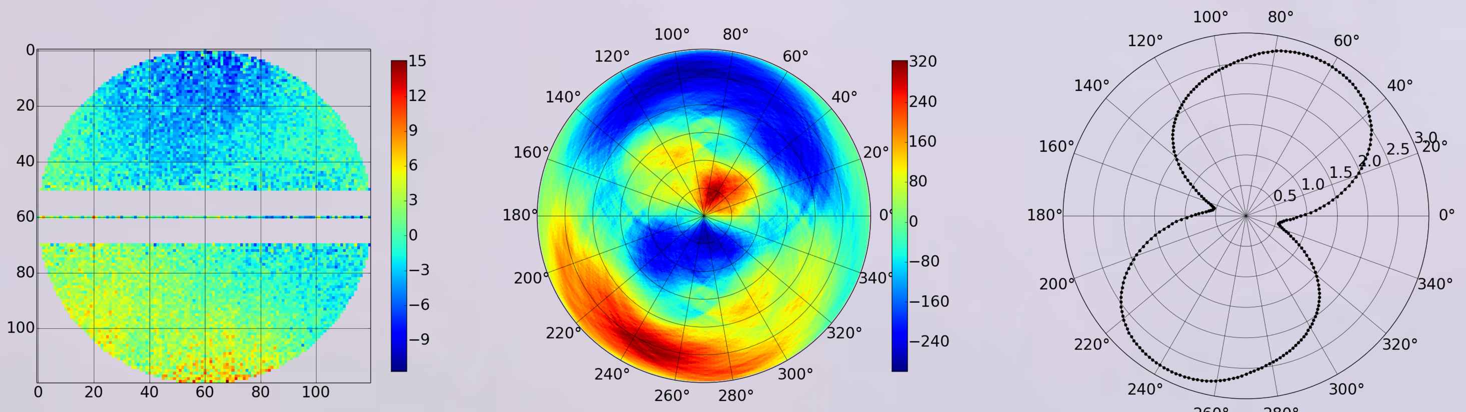
RESULTS

- The model « reality » exhibits more energy in NW/SE direction: « plane wave amplitude » equivalent is 30% higher in this direction (F5)
- The AVISO reconstruction has less energy (unresolved scales) than the nature run in all directions. The reconstruction is closer to the reality in the direction of altimeter tracks There is less energy in the direction not observed by altimeters (F5, F6)
- 90-95% of amplitude is captured in along-track direction (F6)
- Only 75-80% of amplitude is reconstructed in cross-track « blind angle » (F6)

Use case #2: detecting directional errors in SWOT images

METHODOLOGY

- Input data from Dibarboure and Ubelmann (2014)
- Ocean model with submesoscale as 'nature run' (Klein et al, 2008)
- Sample reality as if it was observed by SWOT from all angles
- Simulate some measurements errors: example with roll error which has a predetermined direction (cross-track, 90°)
- Extract hundreds of SWOT snippets (F7) with or without errors
- Compute localized RT and its angular variance for each piece (F7)
- Each RT largely dominated by the oceanic signal
- Compare the average of RT angular variance of error-free and error-corrupted images from SWOT



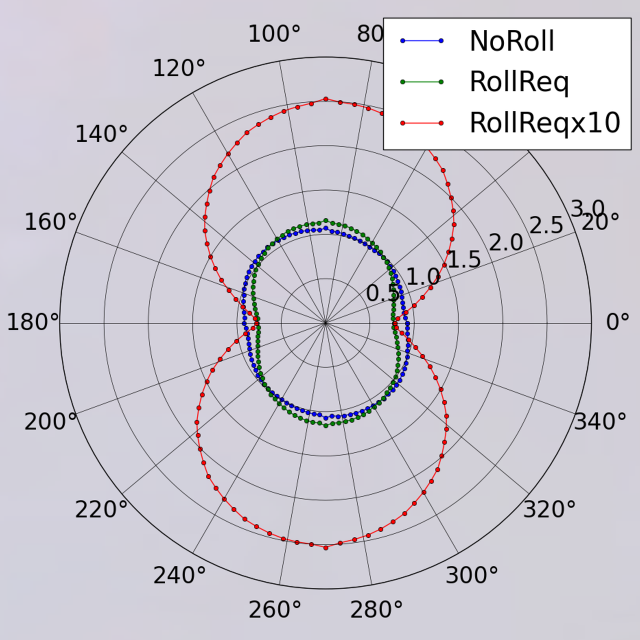
F7: Arbitrary snippet of SWOT image (left), Radon Transform (middle) and angular variance of the RT (right) which are all Dominated by oceanic signal

RESULTS

- With a model reality, roll-free SWOT yields an isotropic RT after averaging a large number of images (F8) taken from all angles
- When roll has 10 times more energy (amplitude x 3) than the requirements, its signature is clearly visible and eight-shaped (F8)
- When roll is within requirements, it is barely detectable (visible here because we have the reference RT)

OUTLOOK

- This method can be used to detect any other directional error in SWOT data (e.g. wet troposphere for a one-horn radiometer)
- A more sophisticated analysis can be used on subsets of data (e.g. geo-graphical boxes, classes of SWH and Wind)



F8: Mean angular variance of the Radon Transform of hundreds SWOT snippets when there is no roll error (blue), when the requirements are met (green) and when the roll variance is ten times above the requirements (red)

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