The Resolution Capabilities of Geostrophic Velocity, Relative Vorticity and Ekman Pumping Fields Estimated from Noisy SWOT Observations of SSH

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Overview

- Determine the RMS error of SWOT estimates of SSH.
- Investigate the scale dependencies of the dynamical fields inferred from a high-resolution model of the California Current System (CCS).
- Determine the effects of SWOT measurement errors on the resolution limitations of dynamical fields constructed from SWOT data.

Note: This analysis does not consider the effects of sampling errors from the limited swath width and revisit time interval of SWOT.

An objective of the SWOT Mission
is to observe submesoscale variability.

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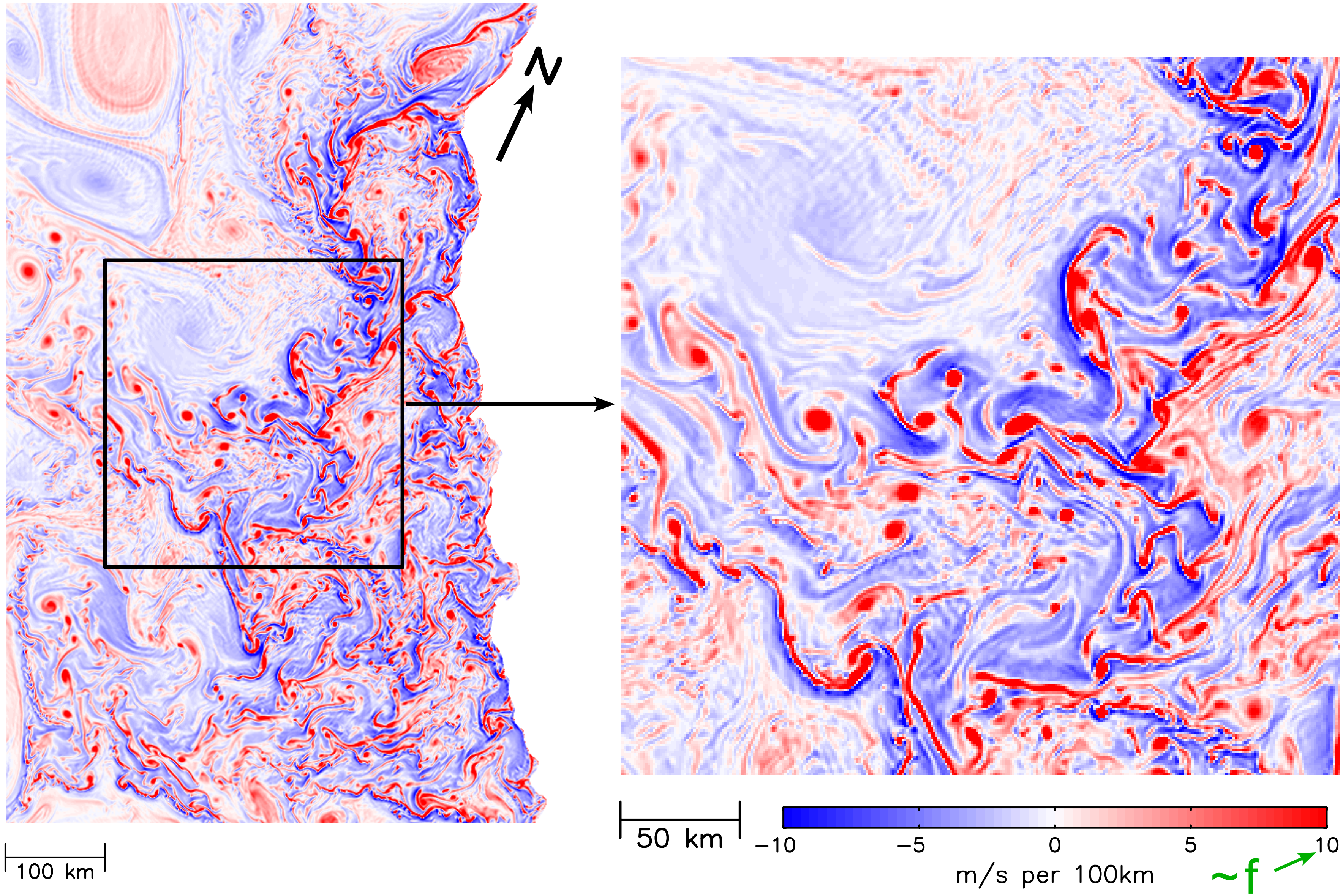
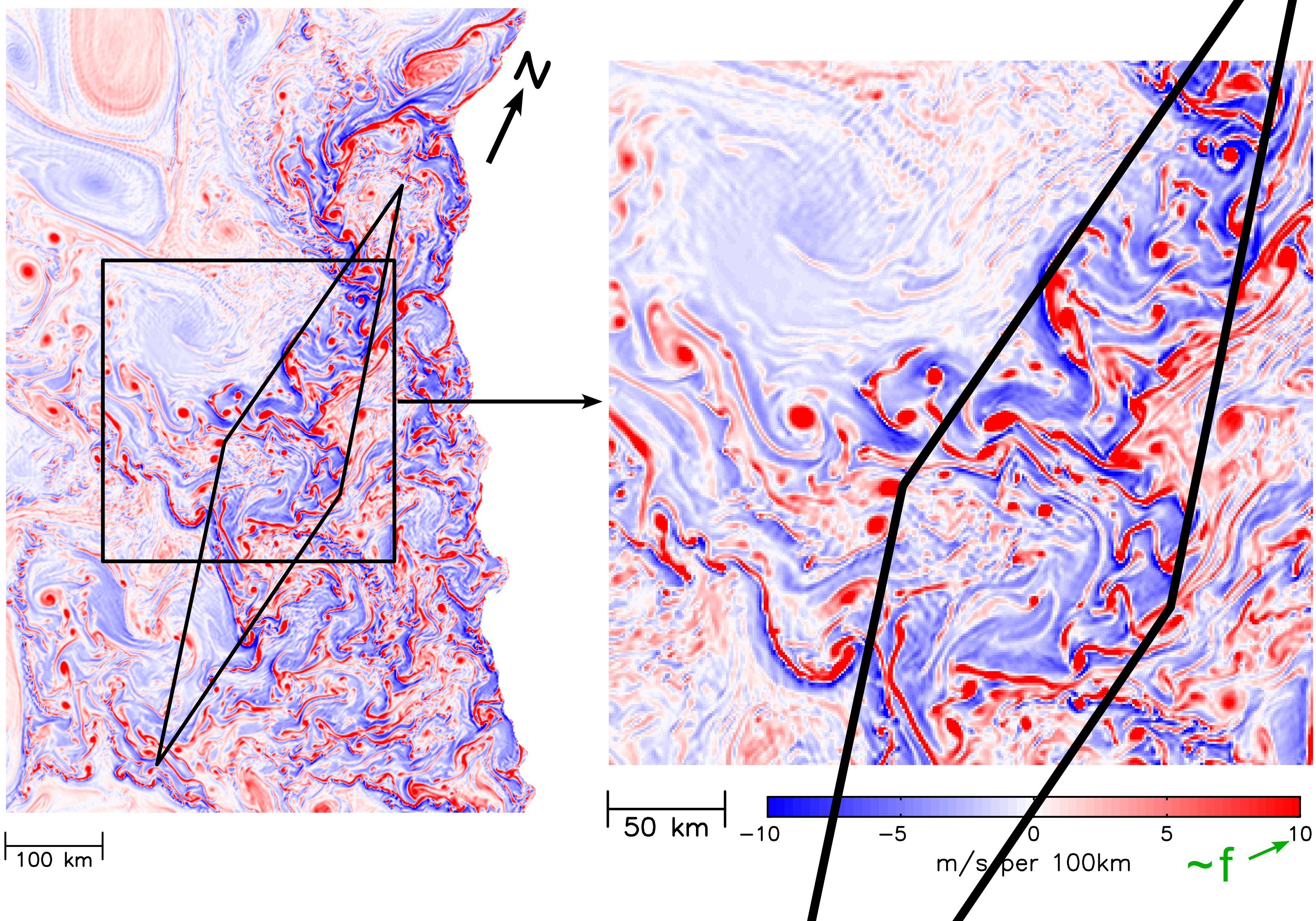
What does submesoscale variability look like in geostrophic velocity and SSH compared with relative vorticity?

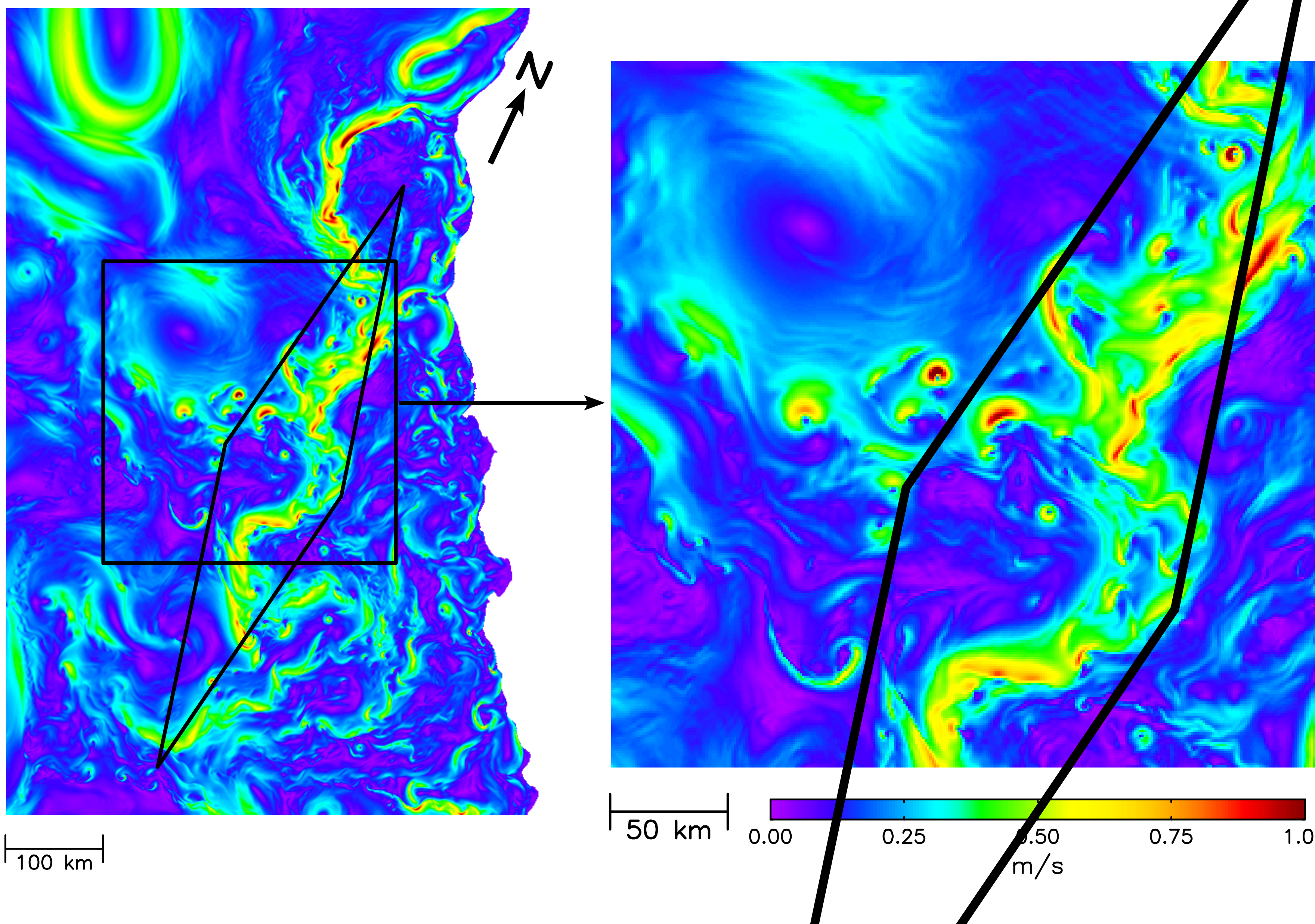
Geostrophic Relative Vorticity from a Model of the CCS with 0.5 km Grid, Smoothed to 1 km Resolution



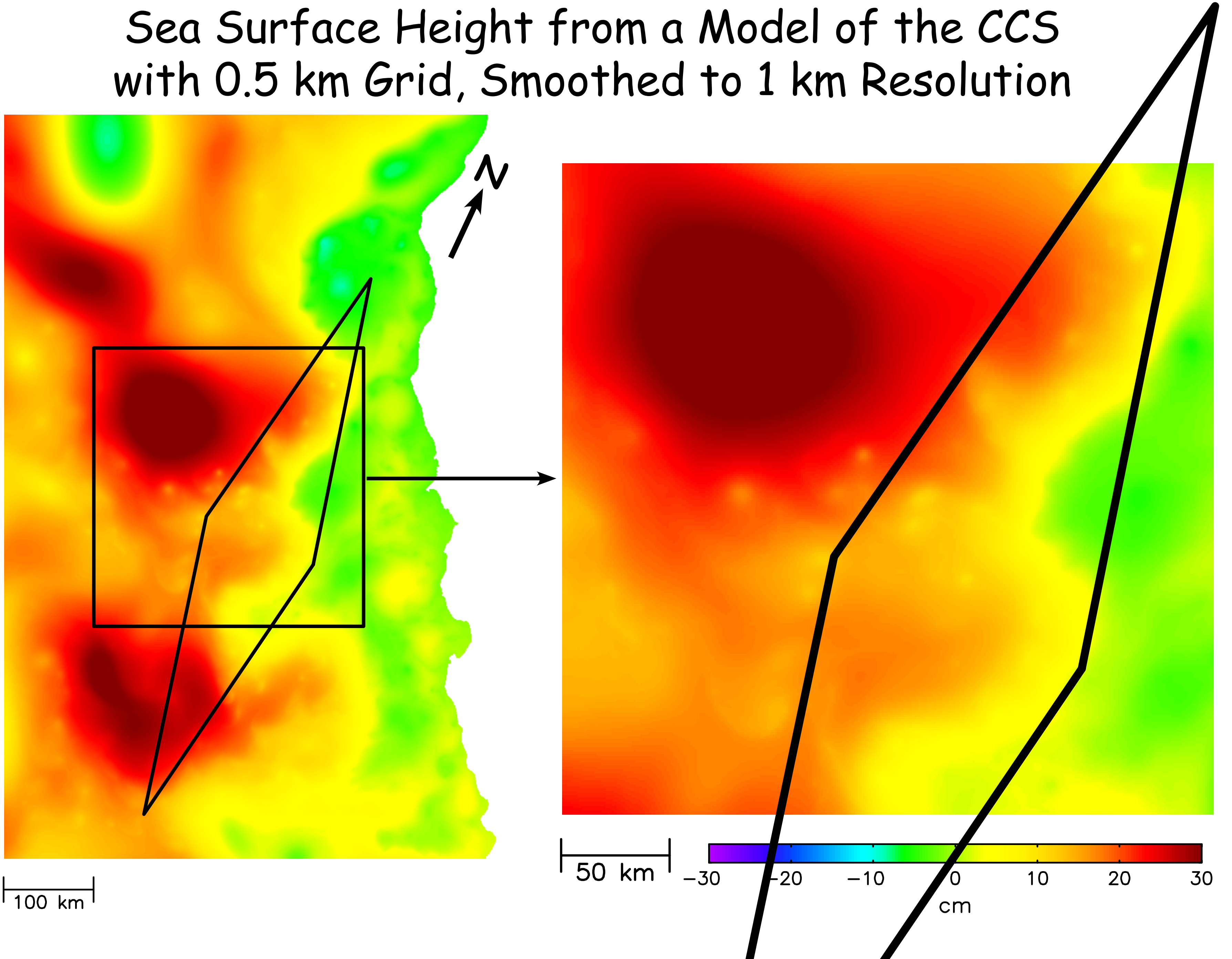
Geostrophic Relative Vorticity from a Model of the CCS with 0.5 km Grid, Smoothed to 1 km Resolution



Geostrophic Speed from a Model of the CCS with 0.5 km Grid, Smoothed to 1 km Resolution

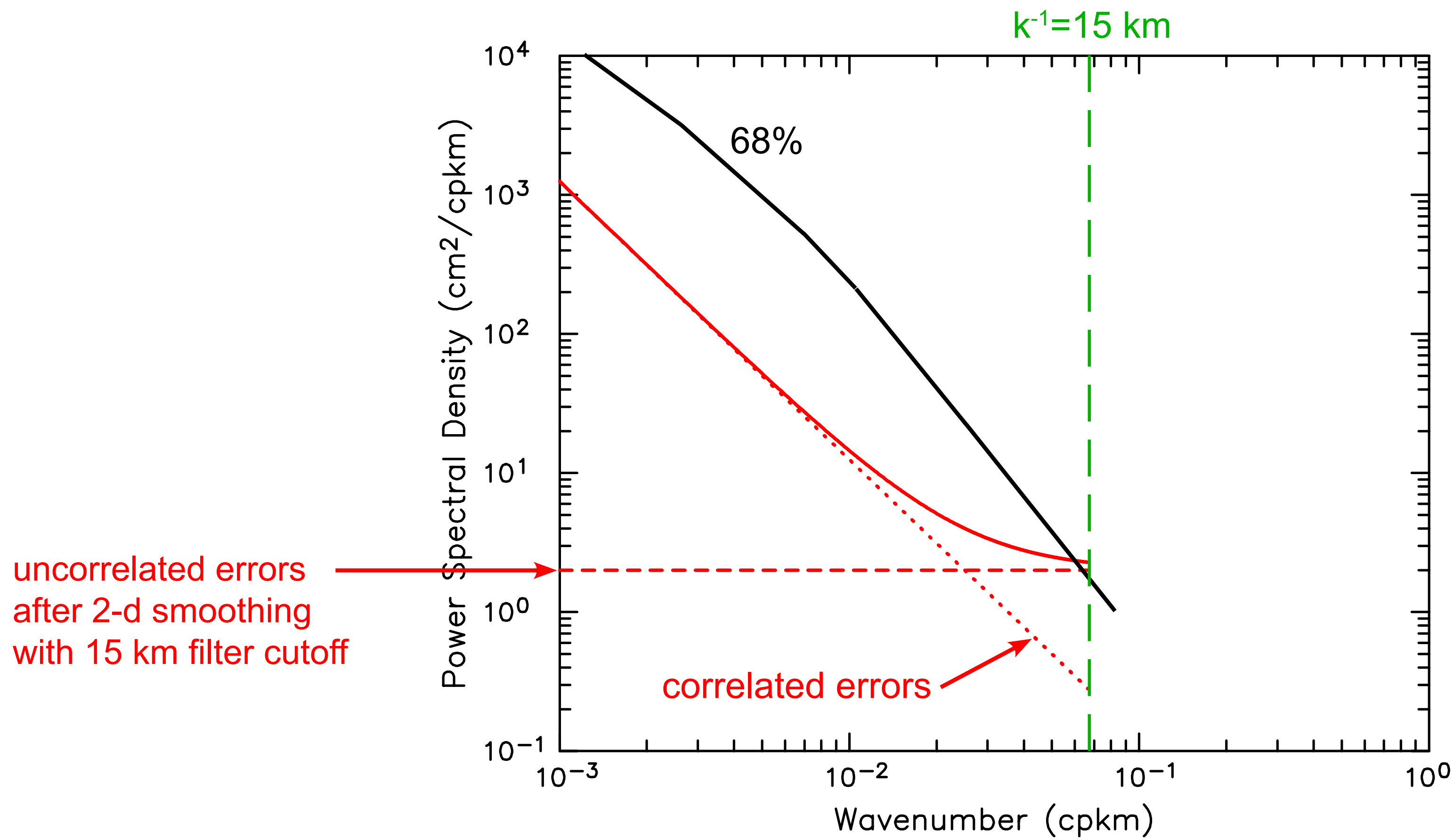


Sea Surface Height from a Model of the CCS with 0.5 km Grid, Smoothed to 1 km Resolution



SWOT Science Requirement for SSH Measurement Accuracy

- The goal of the SWOT mission is to estimate SSH with 2 km wavelength resolution and sufficient accuracy to achieve a signal-to-noise variance ratio greater than 1 for wavelengths of 15–1000 km over 68% of the world ocean.
- To achieve this goal, the spectrum of the white-noise component of SSH measurement errors must be ≤ 2 cm²/cpkm for wavenumbers 1/1000 cpkm ≤ k ≤ 1/15 cpkm.



What is the RMS error of the SWOT onboard processed estimates of SSH that corresponds to a white noise spectrum of 2 cm^2/cpkm over wavenumbers 1/1000 cpkm $\leq k \leq$ 1/15 cpkm?

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The "reverse engineering" procedure:

1. The starting point is the 1-d along-track white spectrum S_{1d} of the errors of onboard processed SWOT estimates of SSH after hypothetical 2-d smoothing with filter cutoff wavelengths of 15 km in both dimensions.
2. Transform S_{1d} into a corresponding 2-d white spectrum S_{2d} using the fact that S_{1d} is the integral of the 2-d white spectrum S_{2d} over all of the wavenumbers in the cross-track direction within the filter pass band.
3. Calculate the variance of the errors of the hypothetically 2-d smoothed onboard processed SWOT estimates of SSH as the integral of S_{2d} over all of the wavenumbers within the filter pass bands in both dimensions.
4. "Undo" the 2-d filtering by dividing the variance of the errors of the hypothetically 2-d smoothed onboard processed SWOT estimates of SSH by the error reduction factor associated with the filter cutoffs.
5. Take the square root of the resulting variance of the unfiltered onboard processed SWOT estimates of SSH to get the RMS measurement error.

For details, see the 33-page "white paper" by Chelton, Samelson and Farrar.

The resulting RMS error of onboard processed SWOT estimates of SSH is .....

(drum roll here)

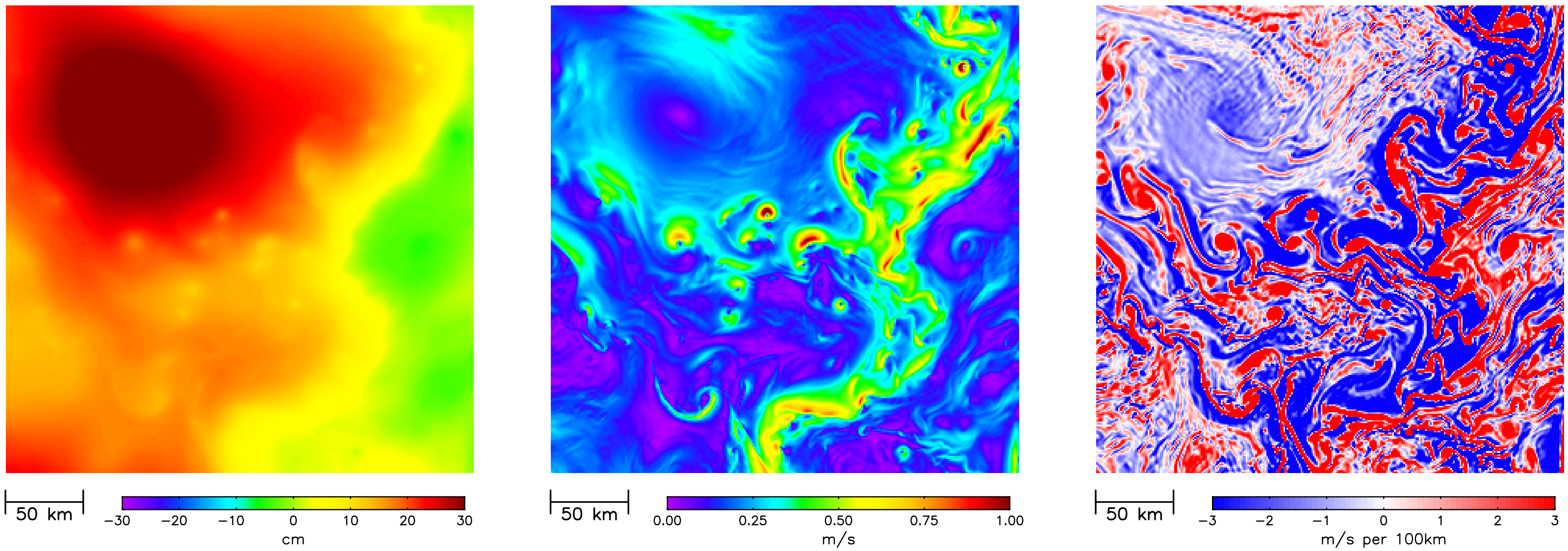
The resulting RMS error of onboard processed SWOT estimates of SSH is .....

(drum roll here)

2.74 cm

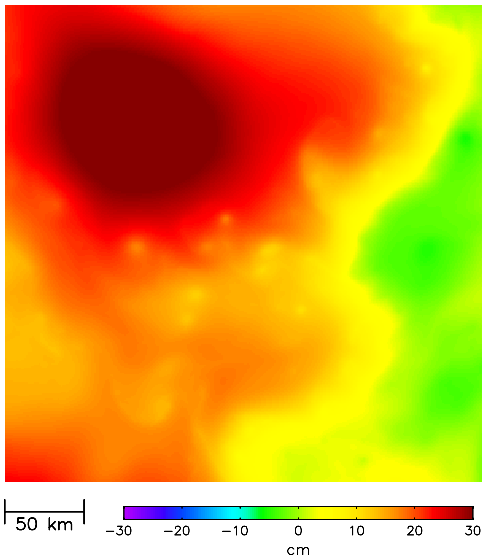
SSH, Geostrophic Speed and Geostrophic Relative Vorticity Unsmoothed with 1 km Resolution

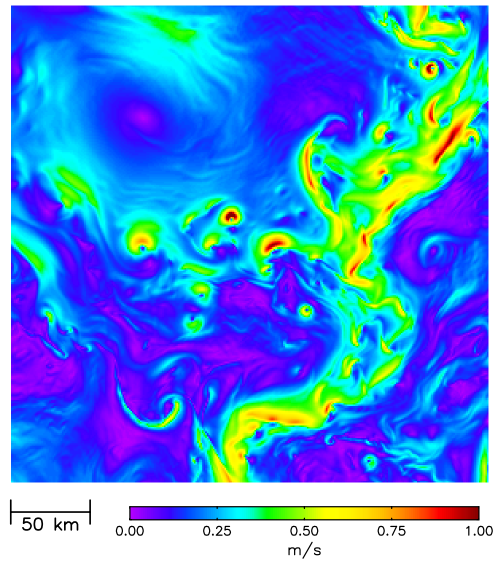
Error Free Measurements

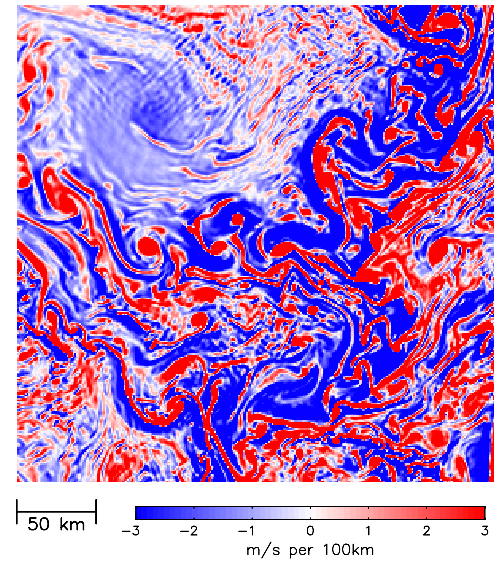


SSH, Geostrophic Speed and Geostrophic Relative Vorticity Unsmoothed with 1 km Resolution

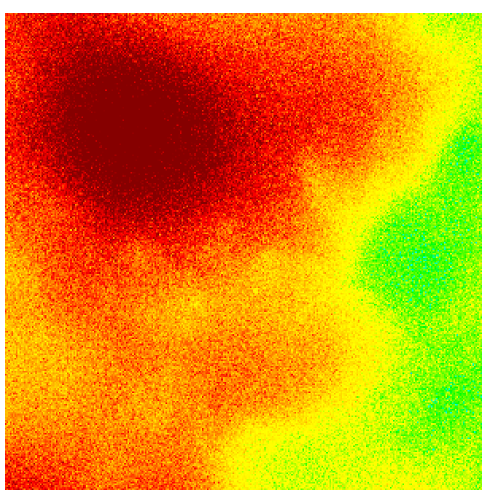
Error Free Measurements





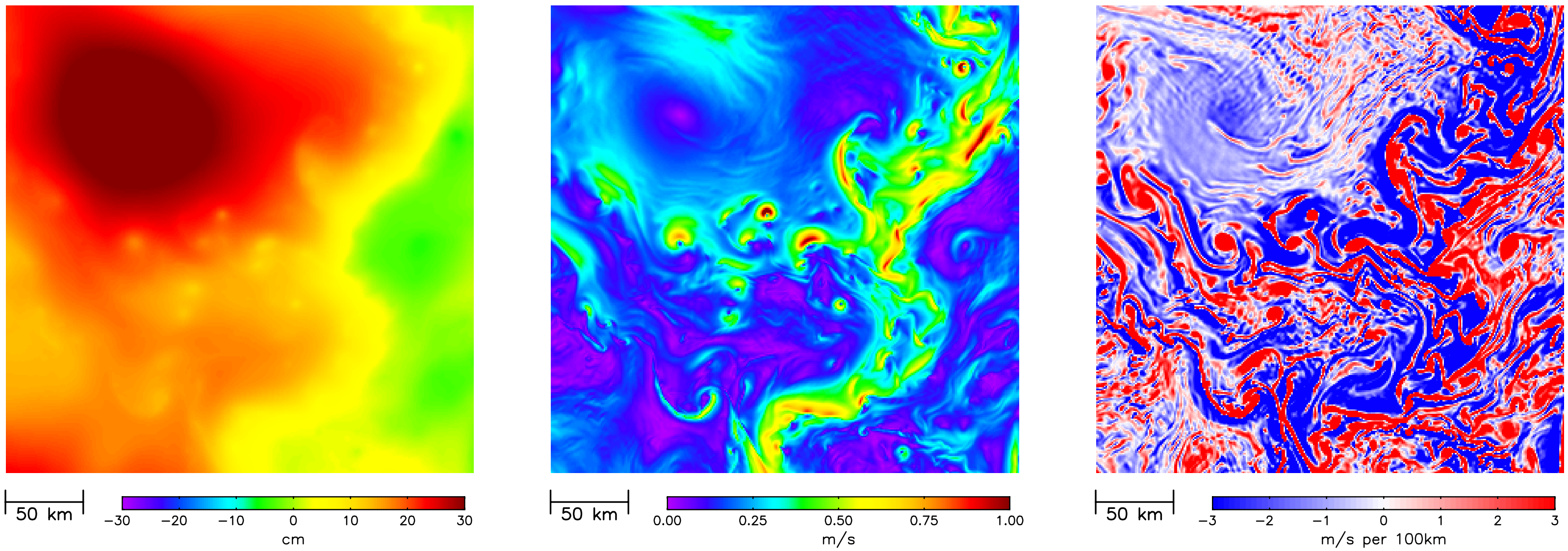


With 2.74 cm RMS Measurement Errors

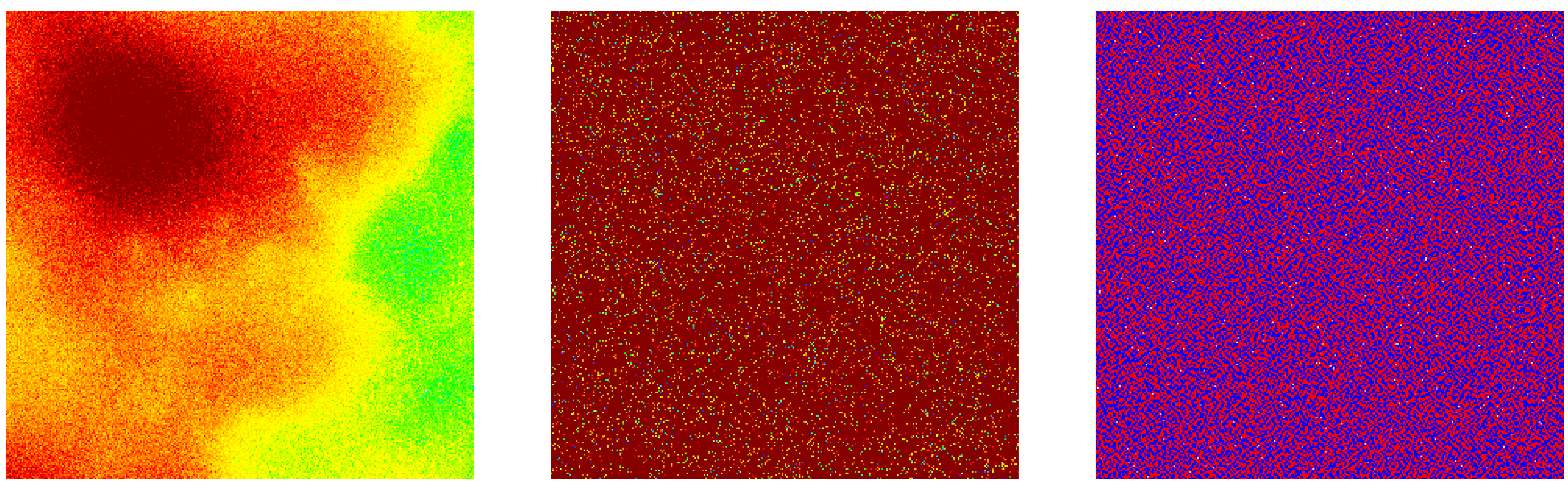


SSH, Geostrophic Speed and Geostrophic Relative Vorticity Unsmoothed with 1 km Resolution

Error Free Measurements

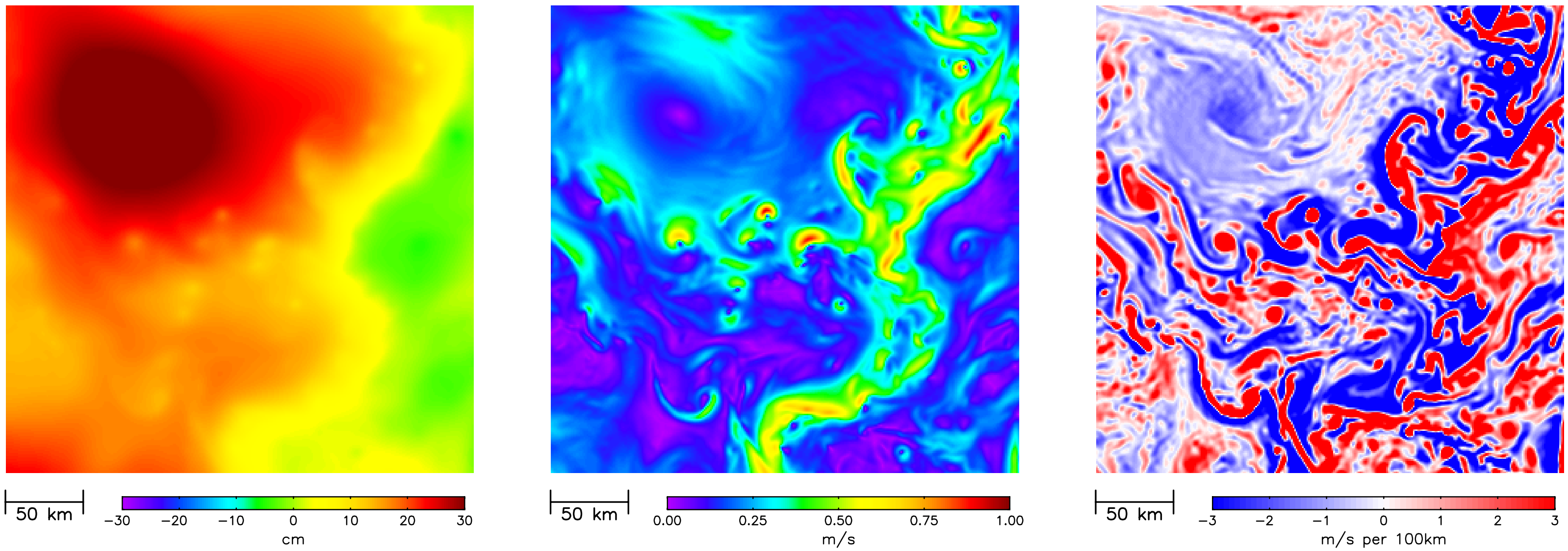


With 2.74 cm RMS Measurement Errors

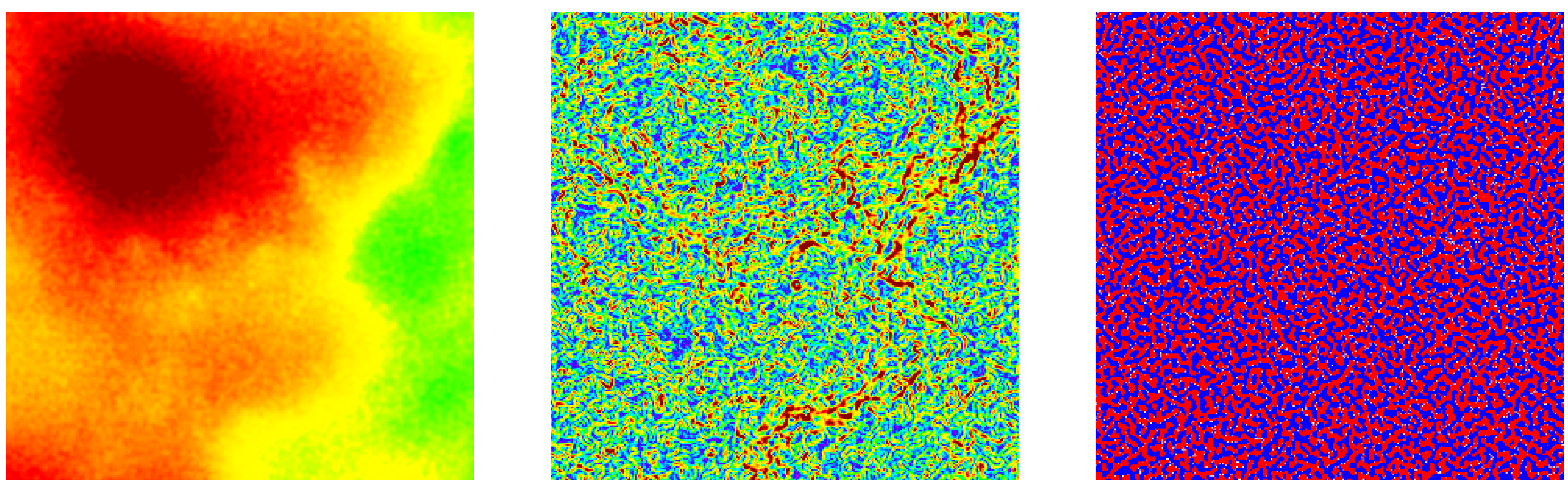


SSH, Geostrophic Speed and Geostrophic Relative Vorticity Smoothed with Filter Cutoff of 10 km

Error Free Measurements

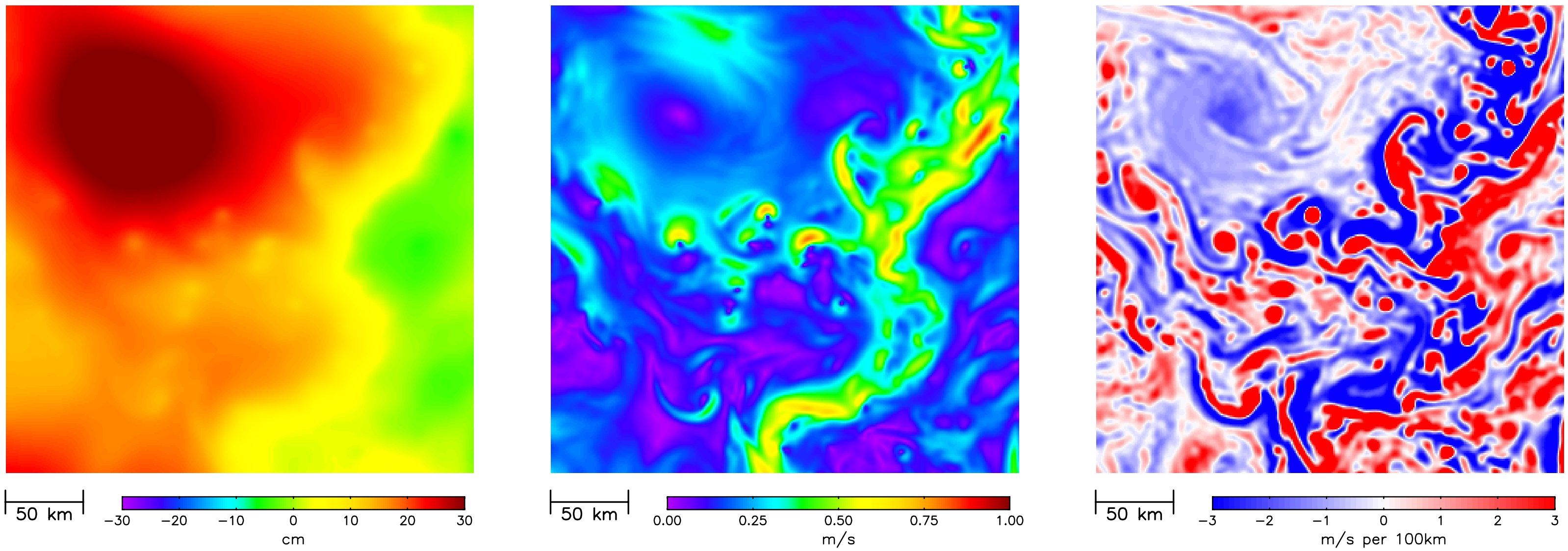


With 2.74 cm RMS Measurement Errors

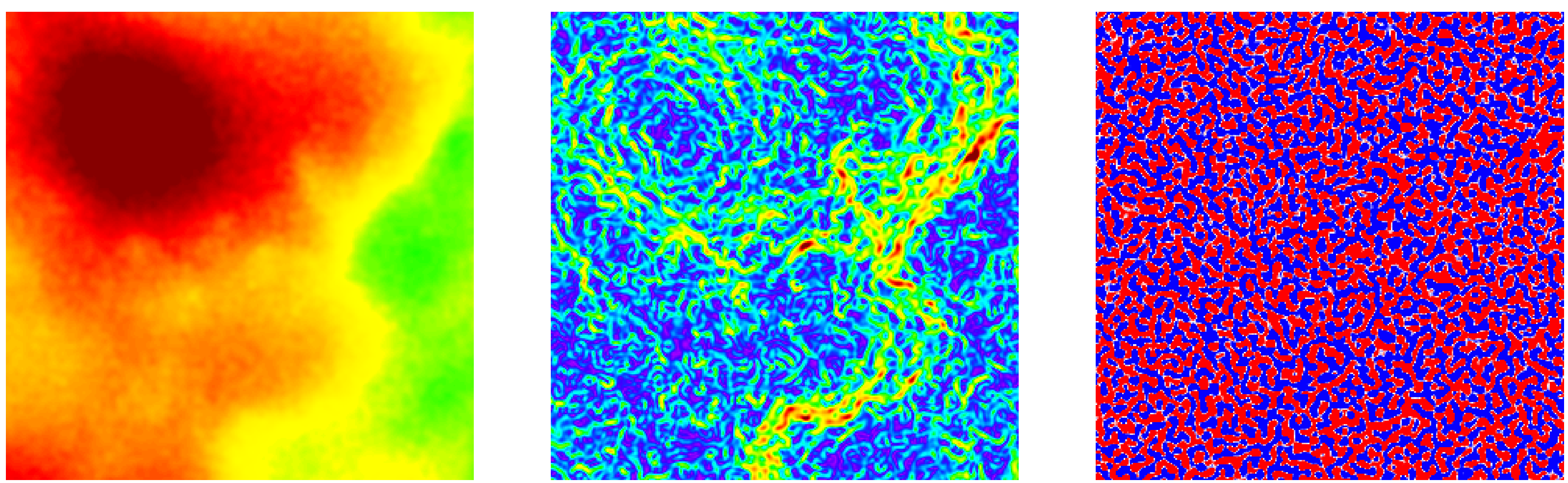


SSH, Geostrophic Speed and Geostrophic Relative Vorticity Smoothed with Filter Cutoff of 15 km

Error Free Measurements

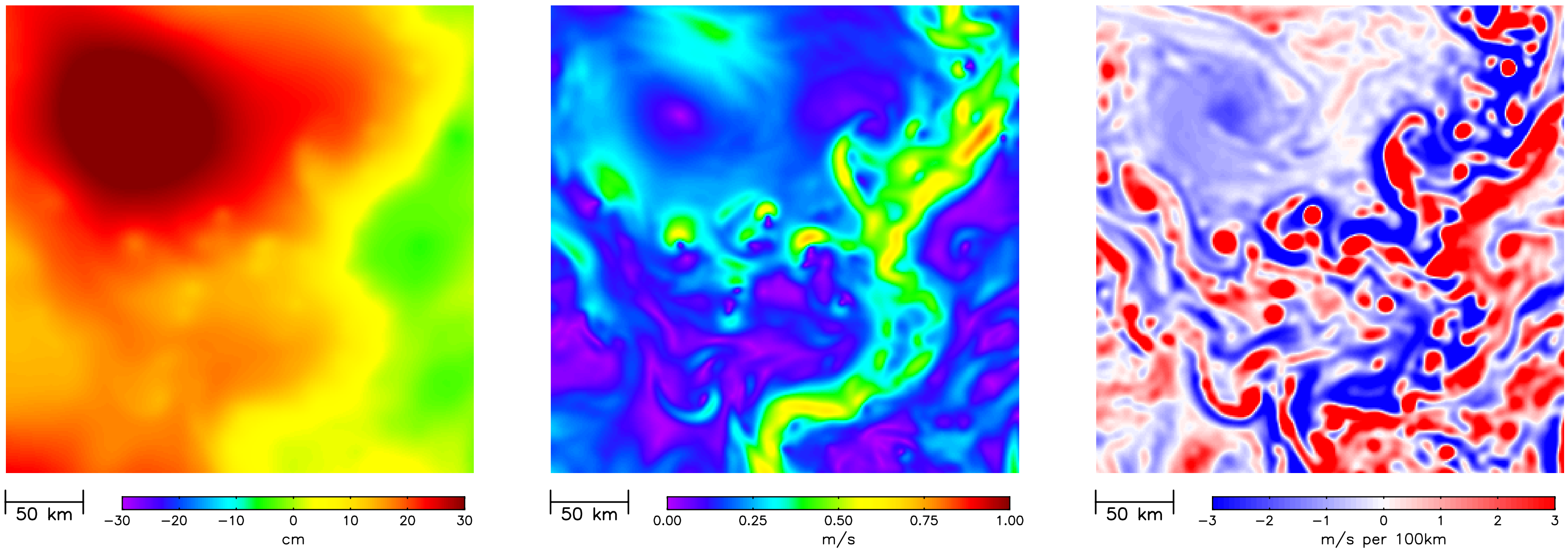


With 2.74 cm RMS Measurement Errors

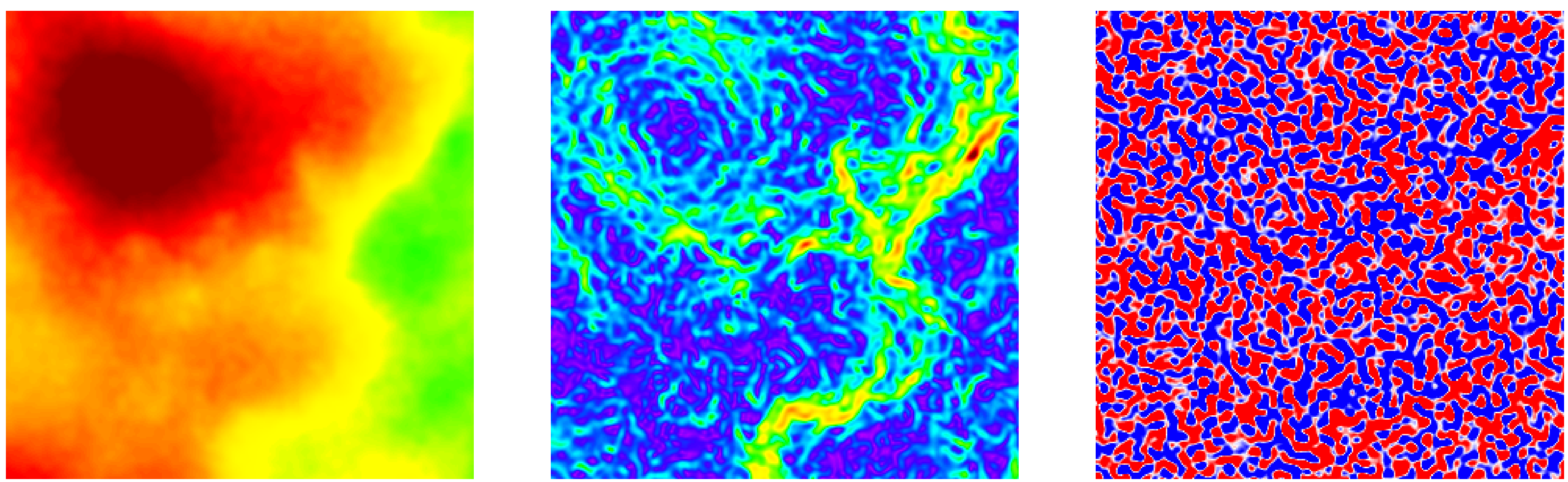


SSH, Geostrophic Speed and Geostrophic Relative Vorticity Smoothed with Filter Cutoff of 20 km

Error Free Measurements

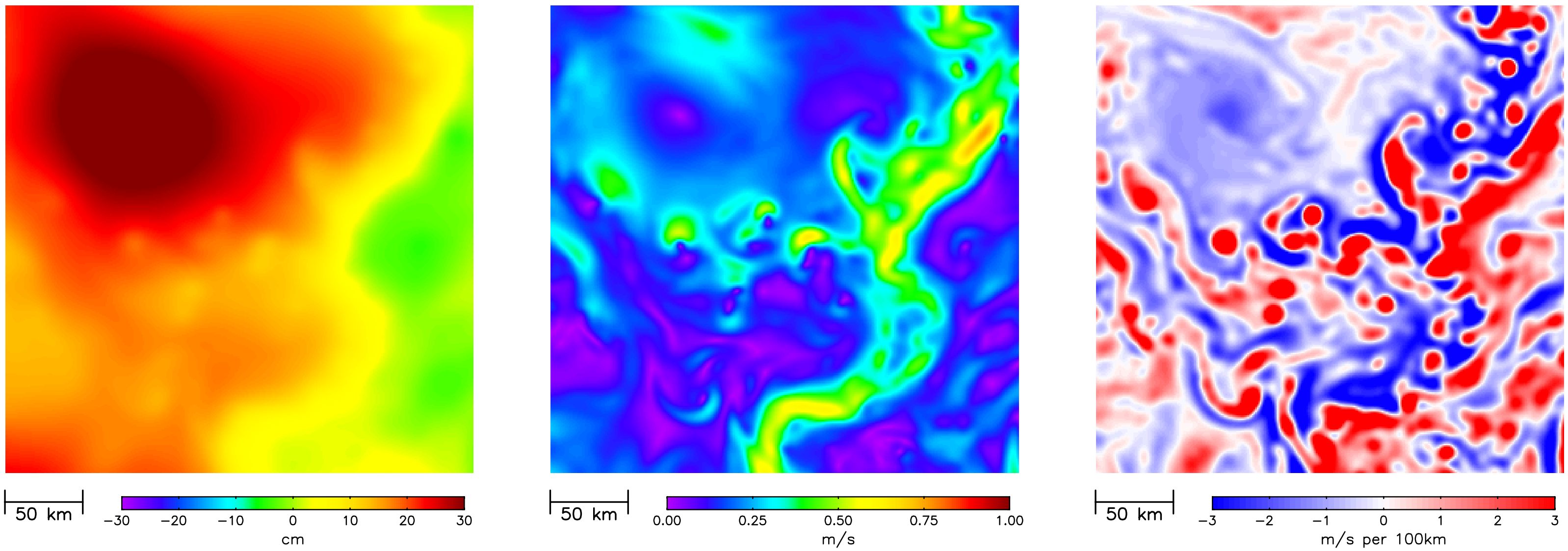


With 2.74 cm RMS Measurement Errors

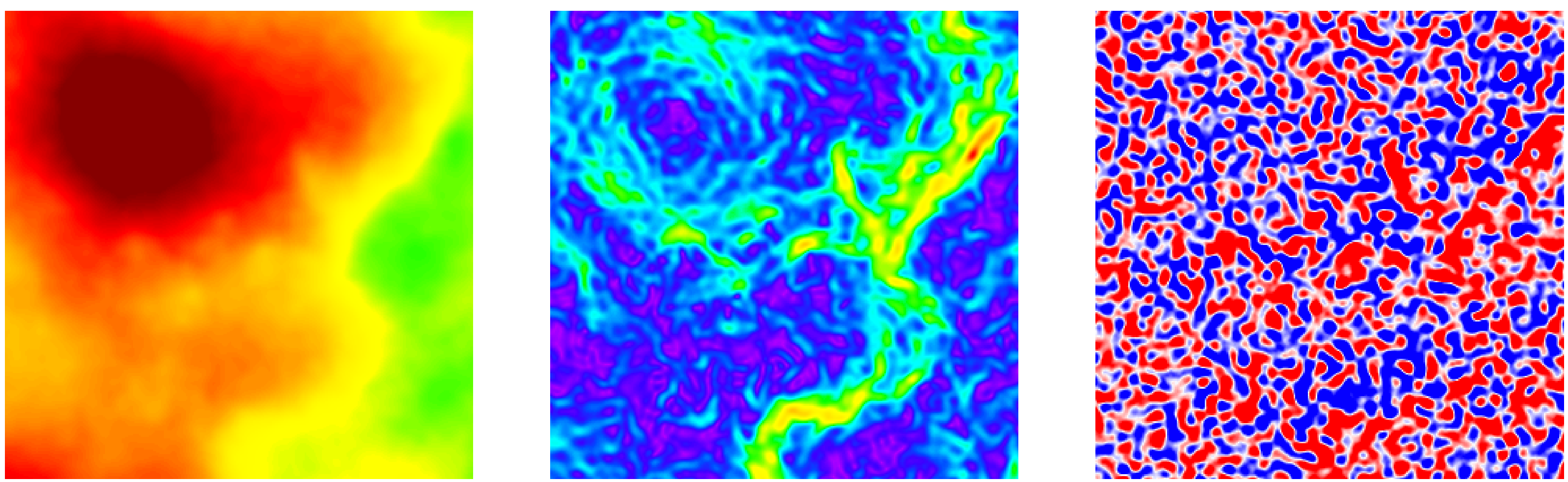


SSH, Geostrophic Speed and Geostrophic Relative Vorticity Smoothed with Filter Cutoff of 25 km

Error Free Measurements

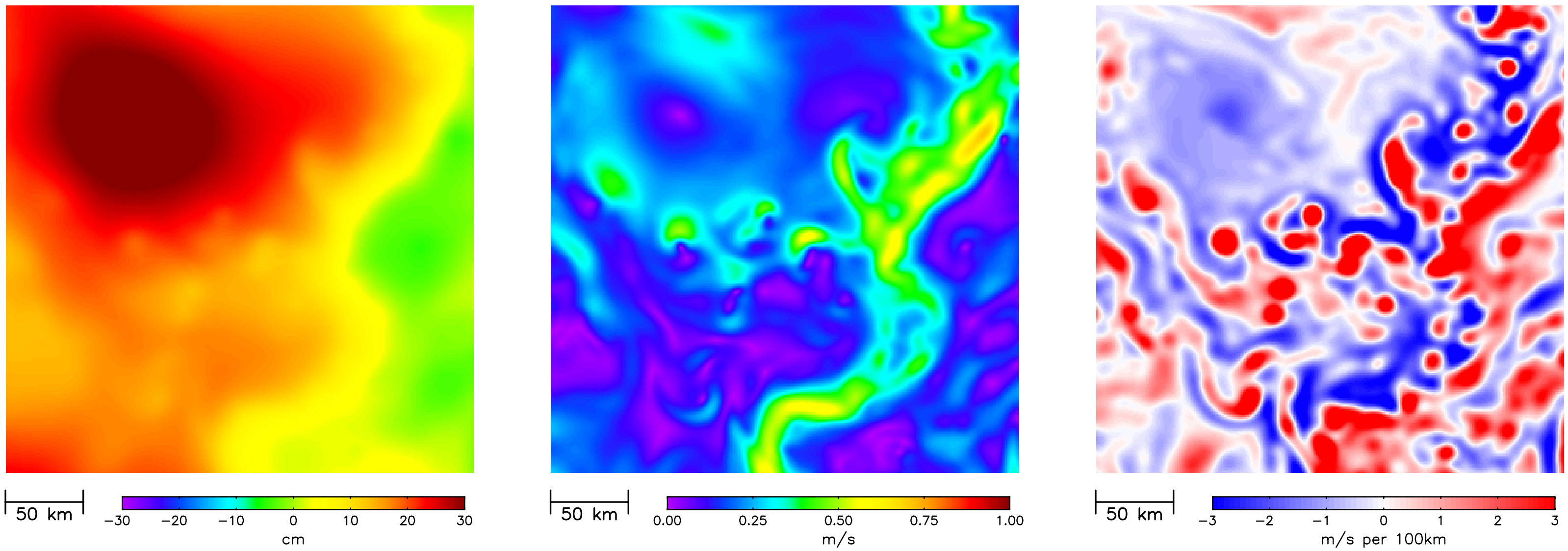


With 2.74 cm RMS Measurement Errors

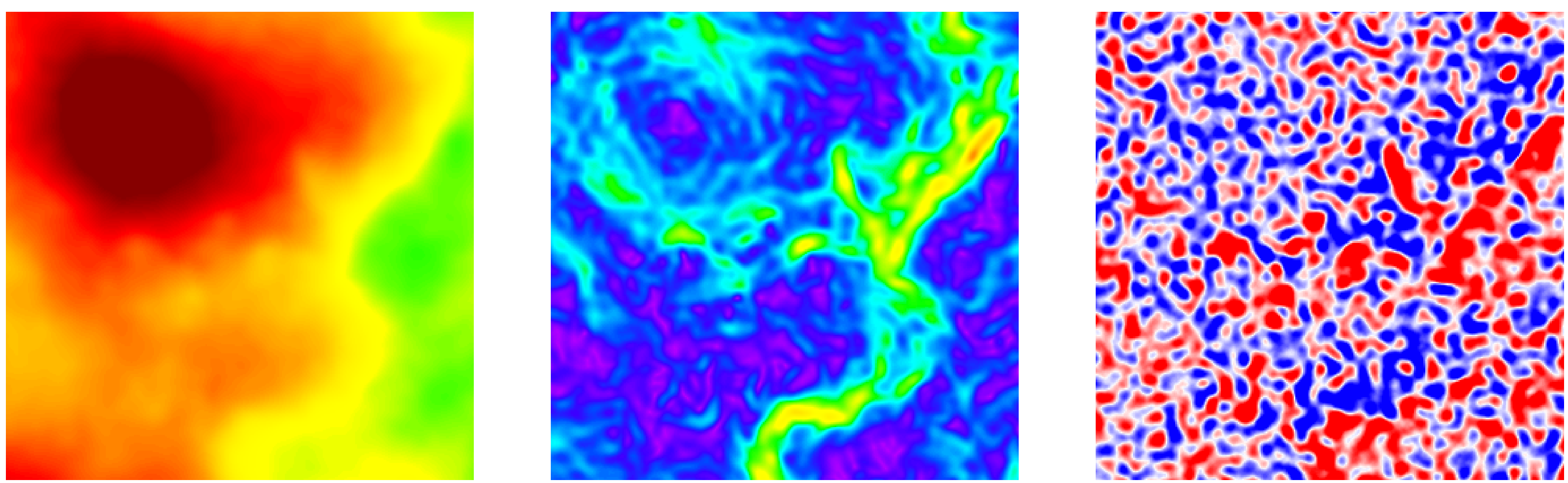


SSH, Geostrophic Speed and Geostrophic Relative Vorticity Smoothed with Filter Cutoff of 30 km

Error Free Measurements

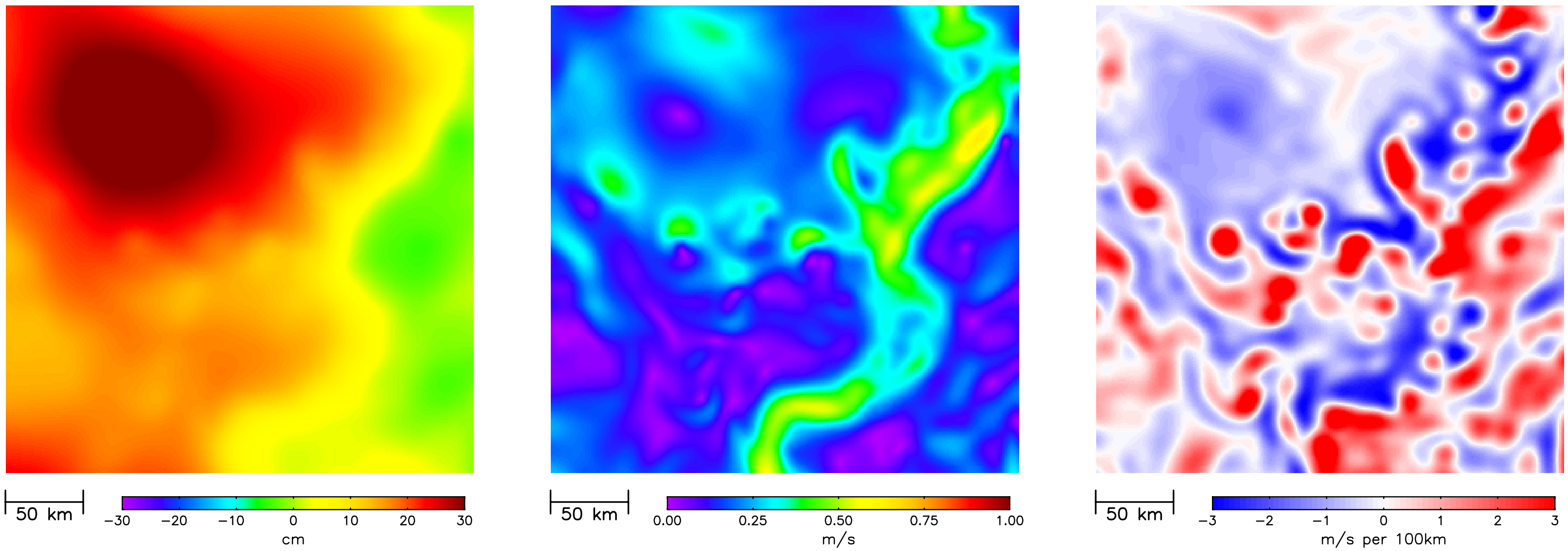


With 2.74 cm RMS Measurement Errors

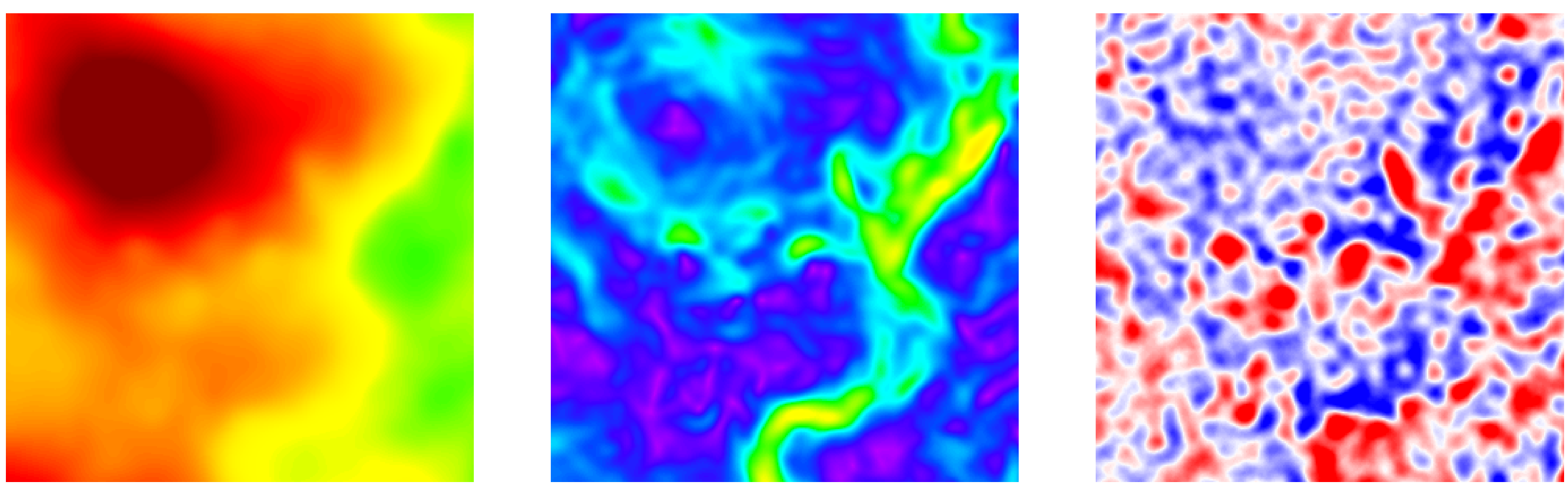


SSH, Geostrophic Speed and Geostrophic Relative Vorticity Smoothed with Filter Cutoff of 40 km

Error Free Measurements

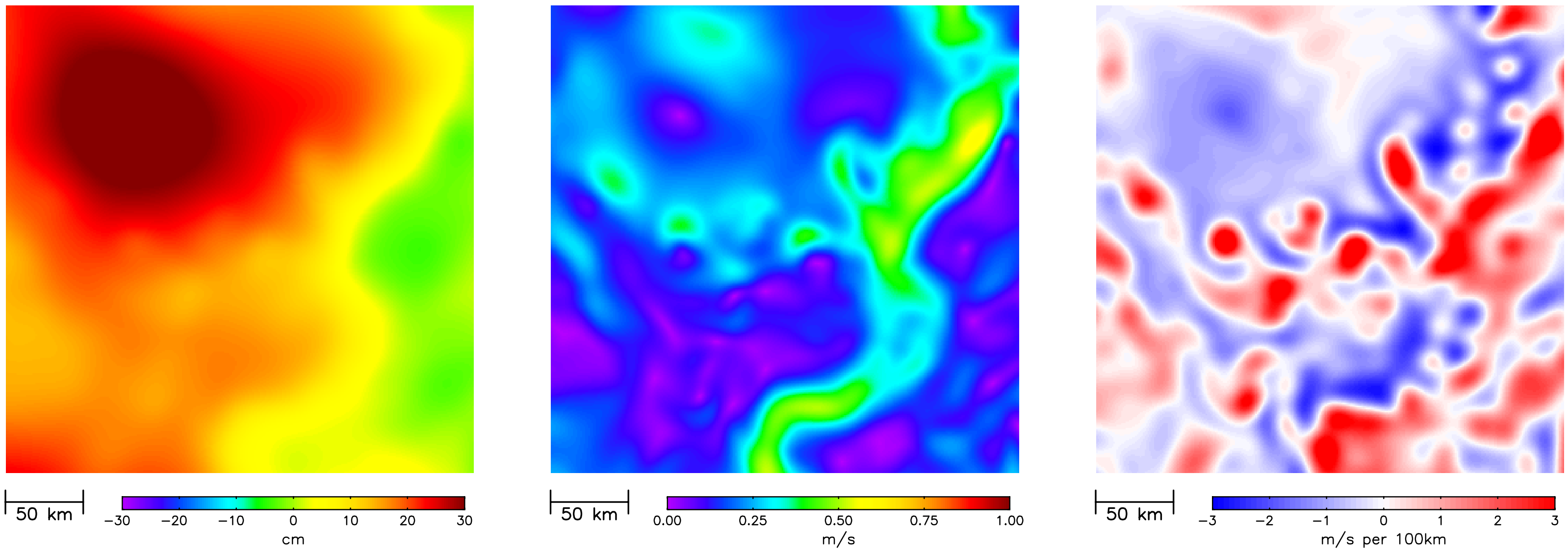


With 2.74 cm RMS Measurement Errors

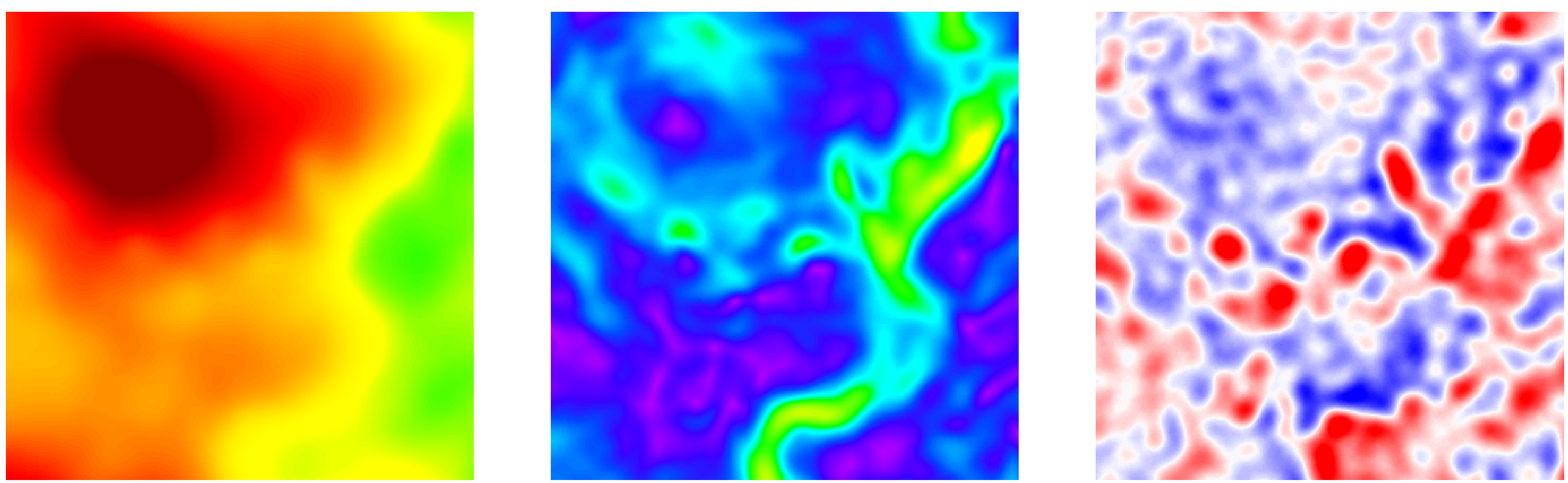


SSH, Geostrophic Speed and Geostrophic Relative Vorticity Smoothed with Filter Cutoff of 50 km

Error Free Measurements

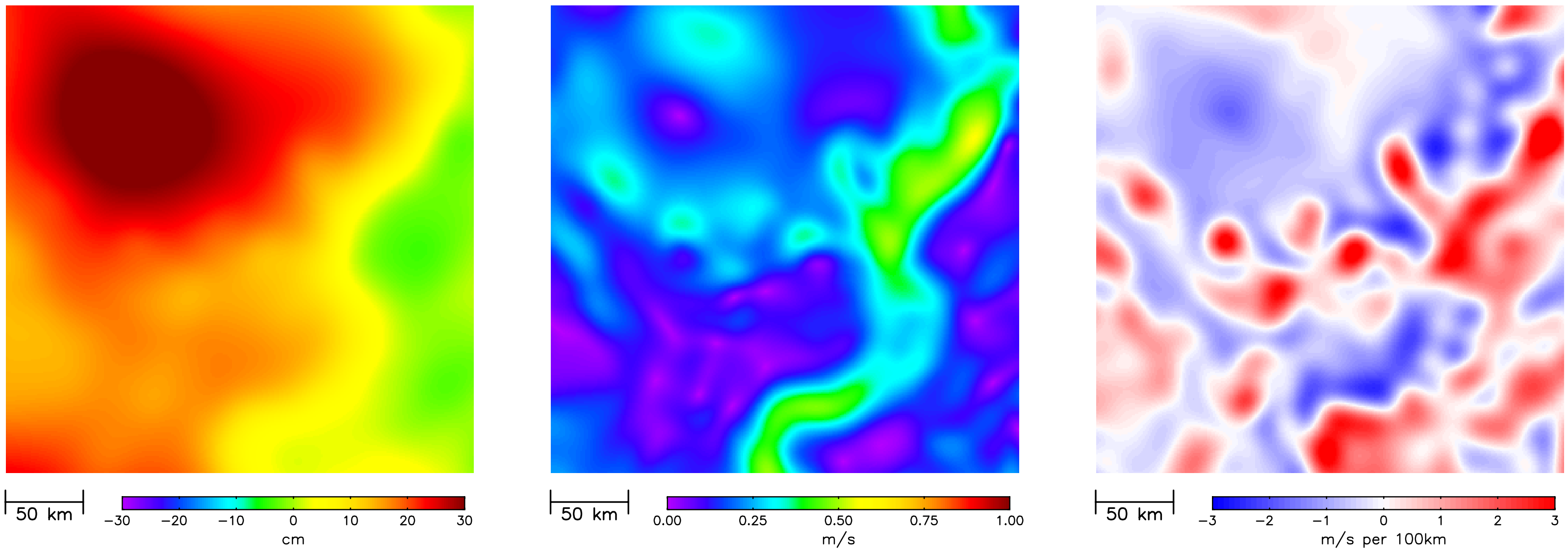


With 2.74 cm RMS Measurement Errors

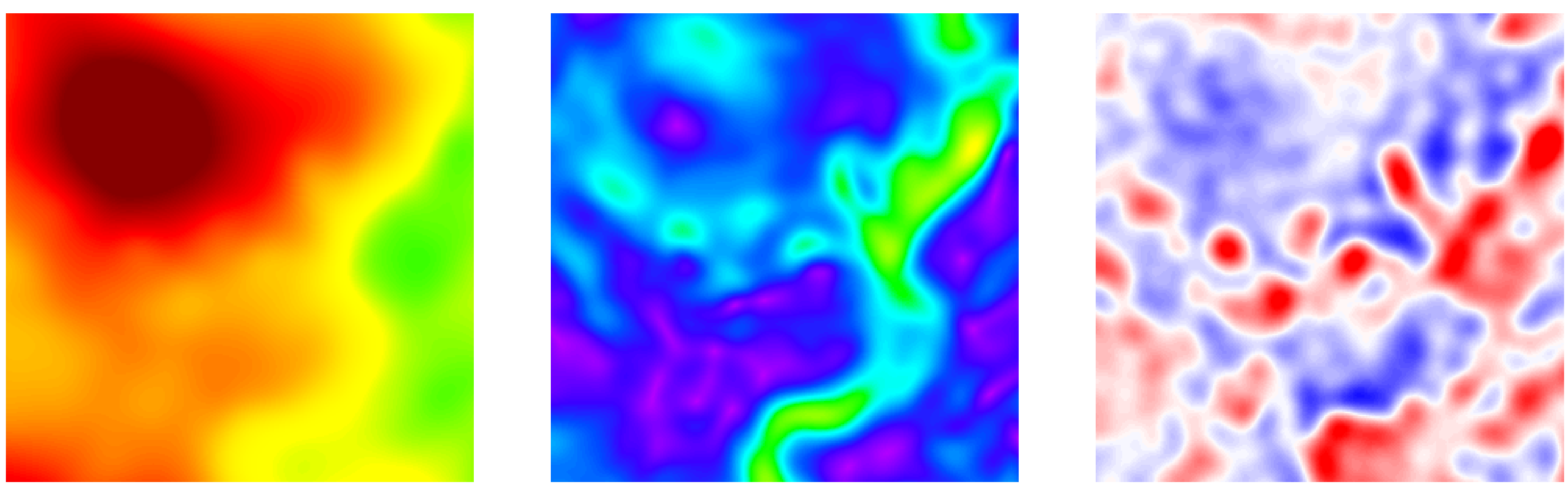


SSH, Geostrophic Speed and Geostrophic Relative Vorticity Smoothed with Filter Cutoff of 60 km

Error Free Measurements

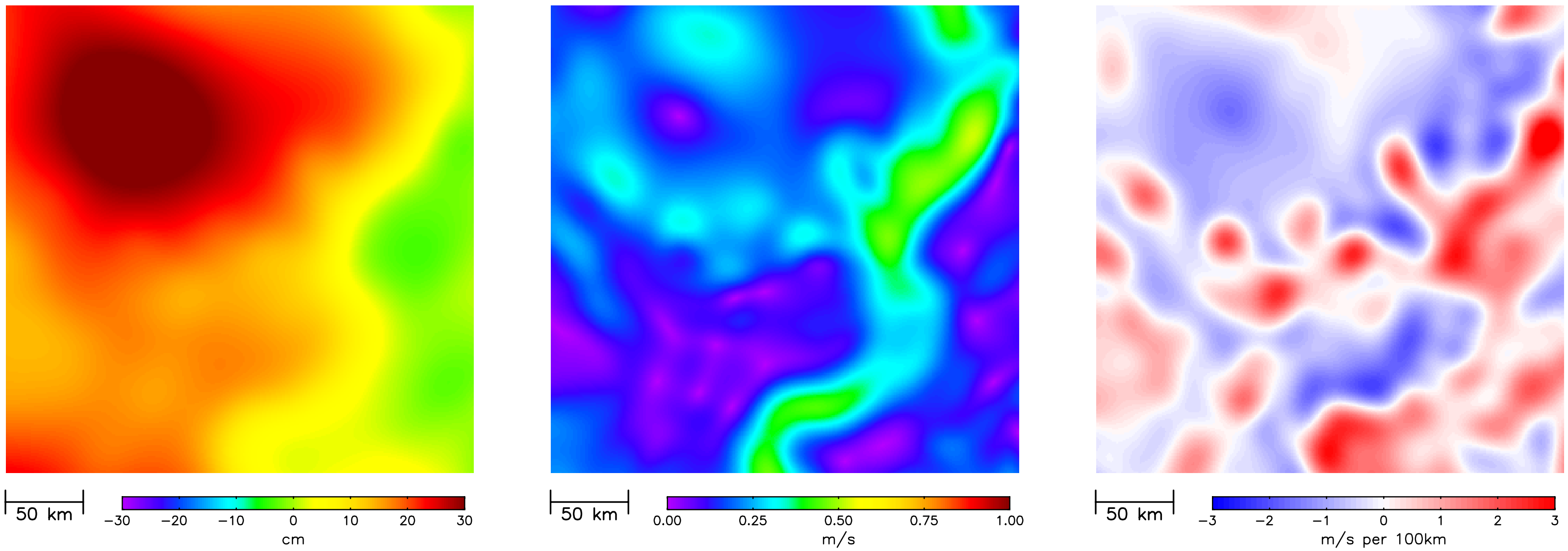


With 2.74 cm RMS Measurement Errors

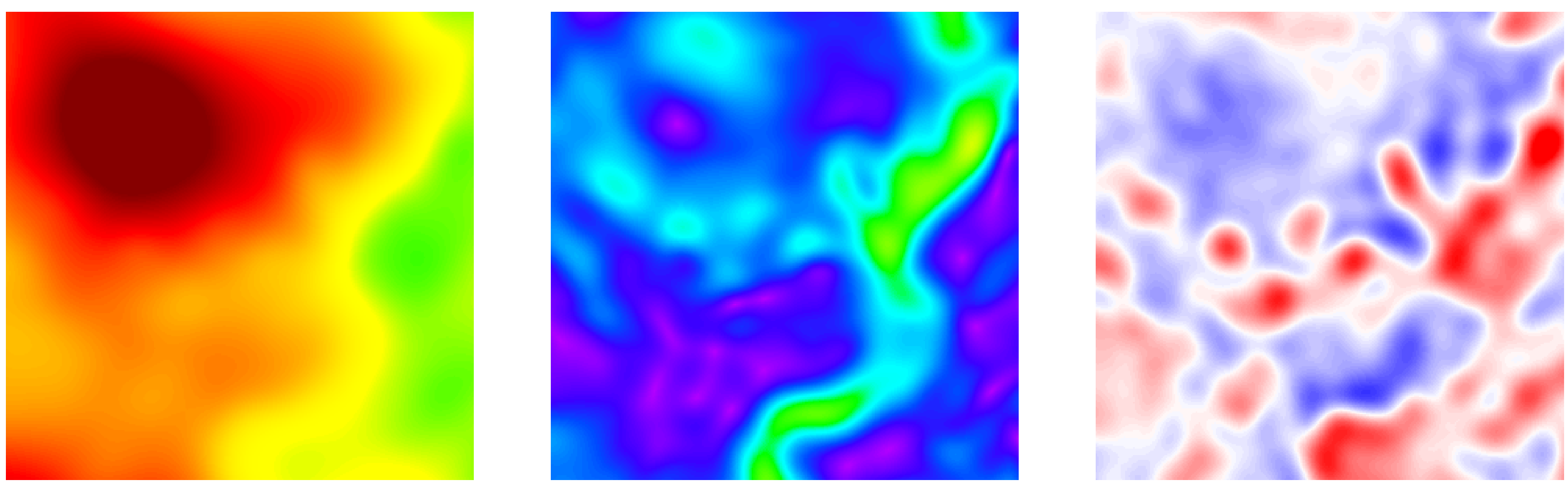


SSH, Geostrophic Speed and Geostrophic Relative Vorticity Smoothed with Filter Cutoff of 70 km

Error Free Measurements

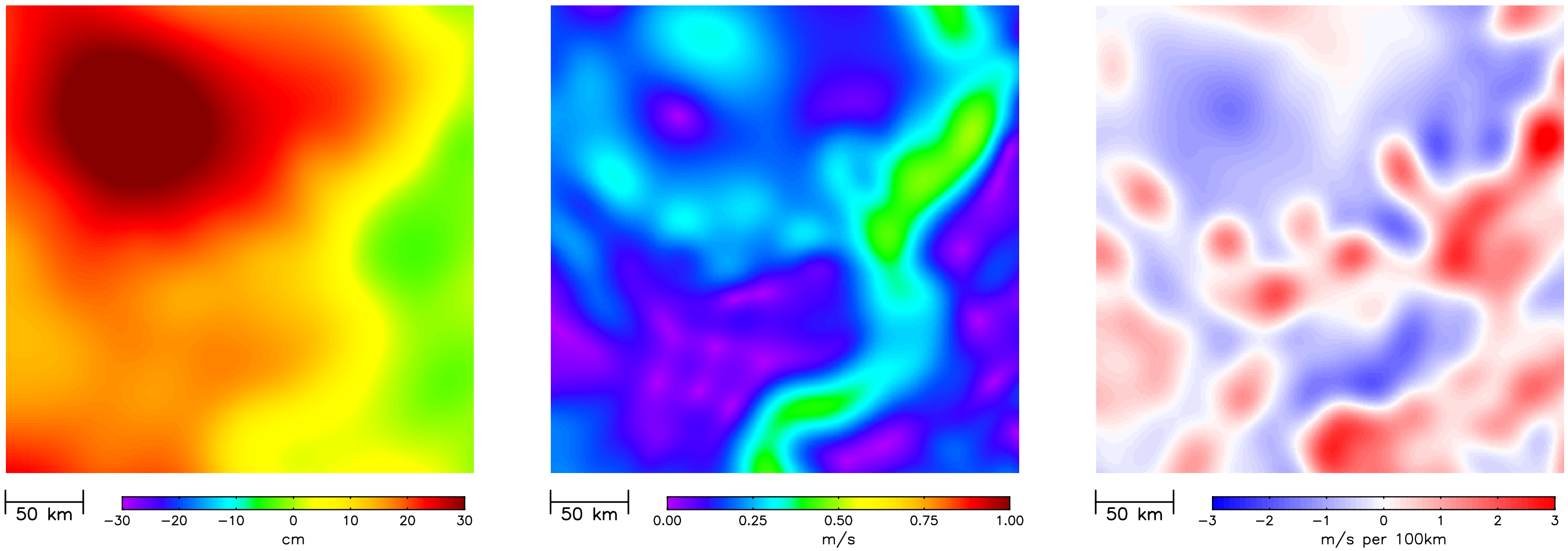


With 2.74 cm RMS Measurement Errors

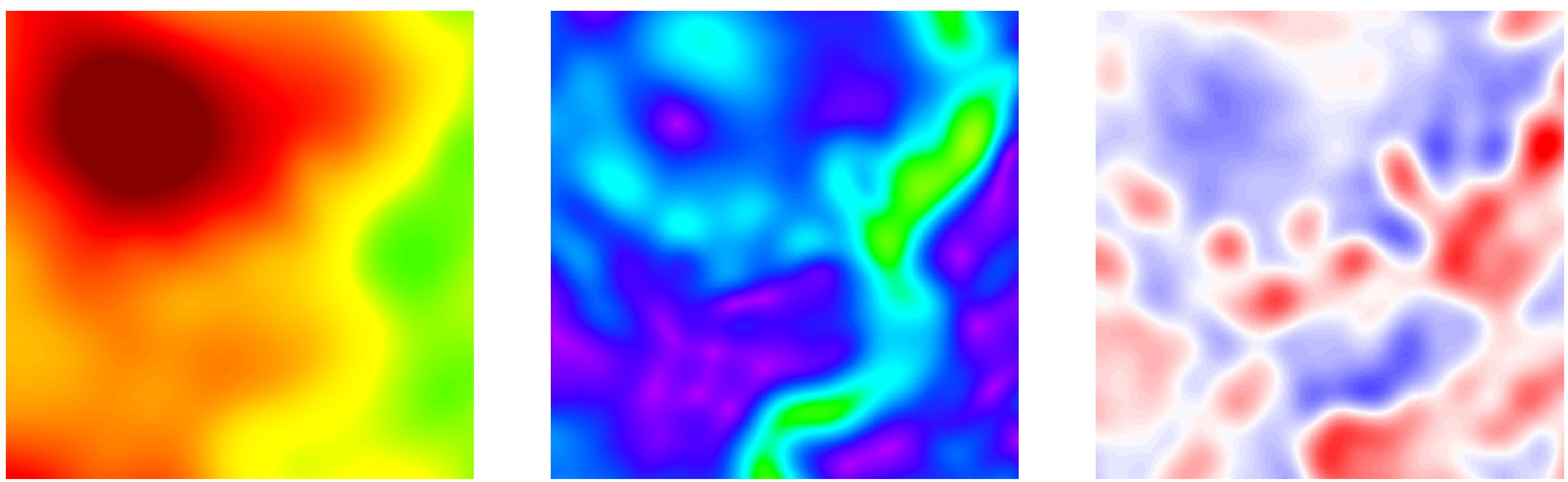


SSH, Geostrophic Speed and Geostrophic Relative Vorticity Smoothed with Filter Cutoff of 80 km

Error Free Measurements

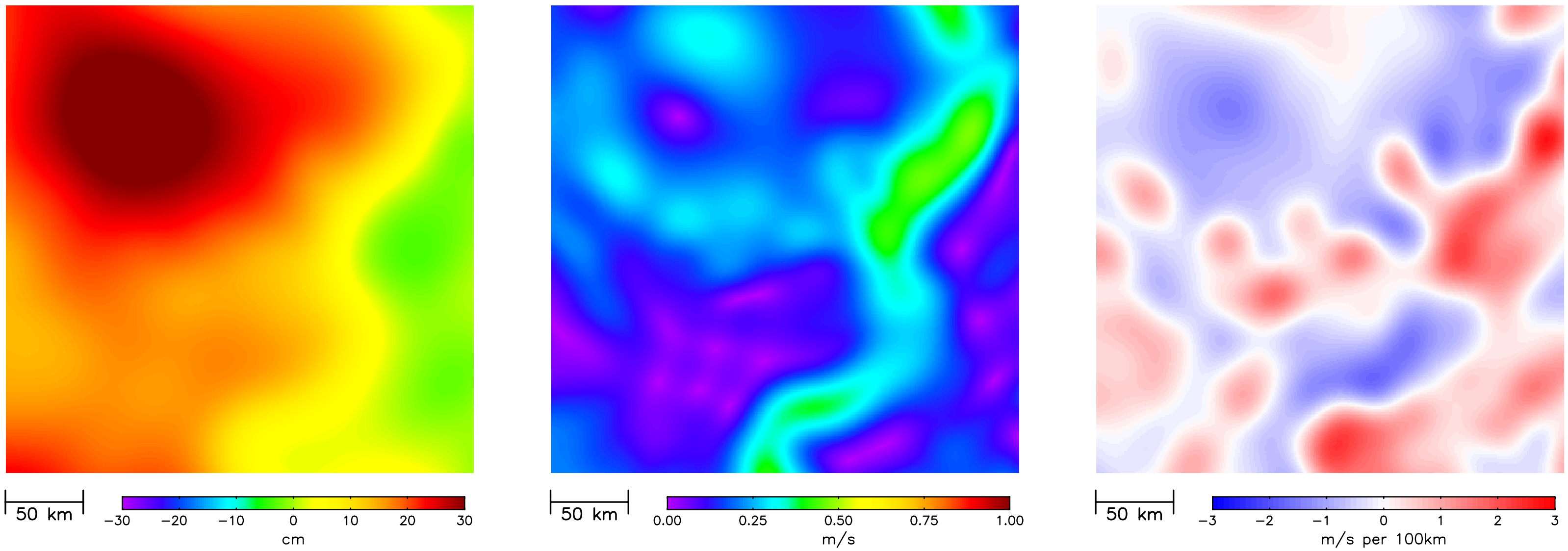


With 2.74 cm RMS Measurement Errors

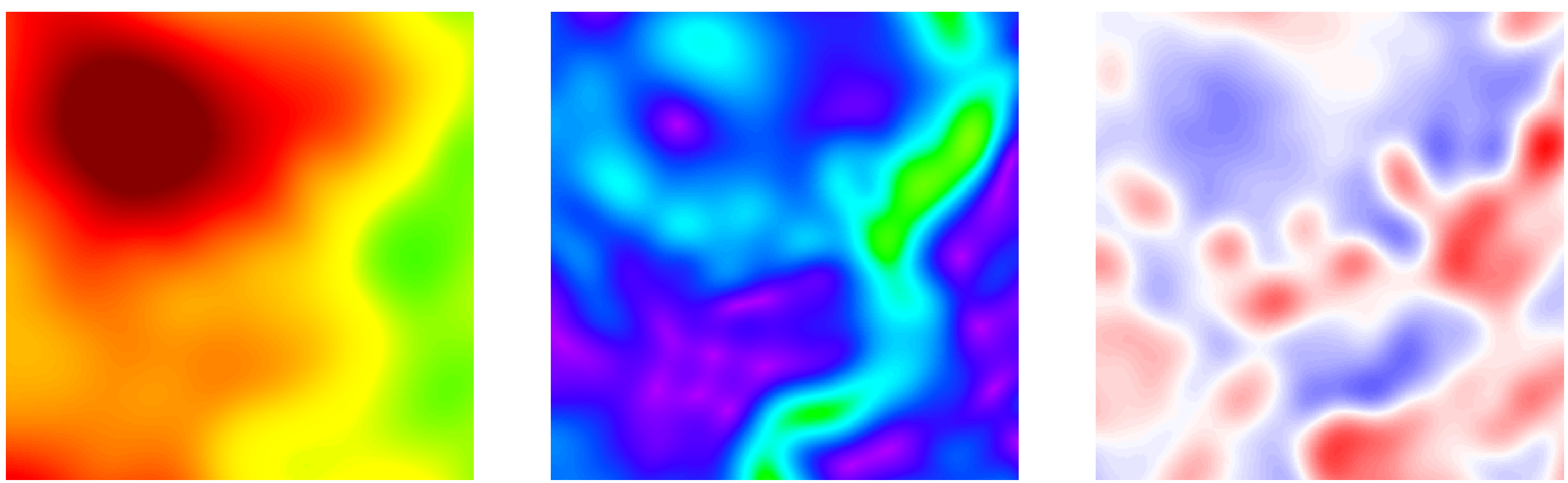


SSH, Geostrophic Speed and Geostrophic Relative Vorticity Smoothed with Filter Cutoff of 90 km

Error Free Measurements

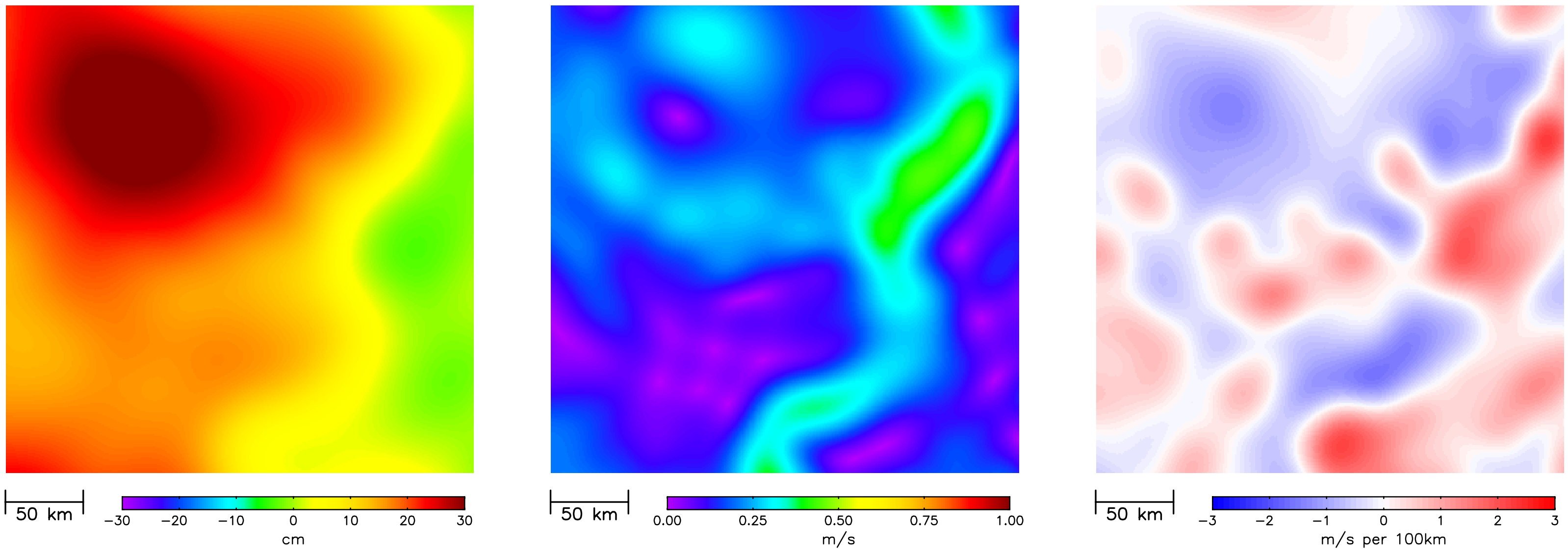


With 2.74 cm RMS Measurement Errors

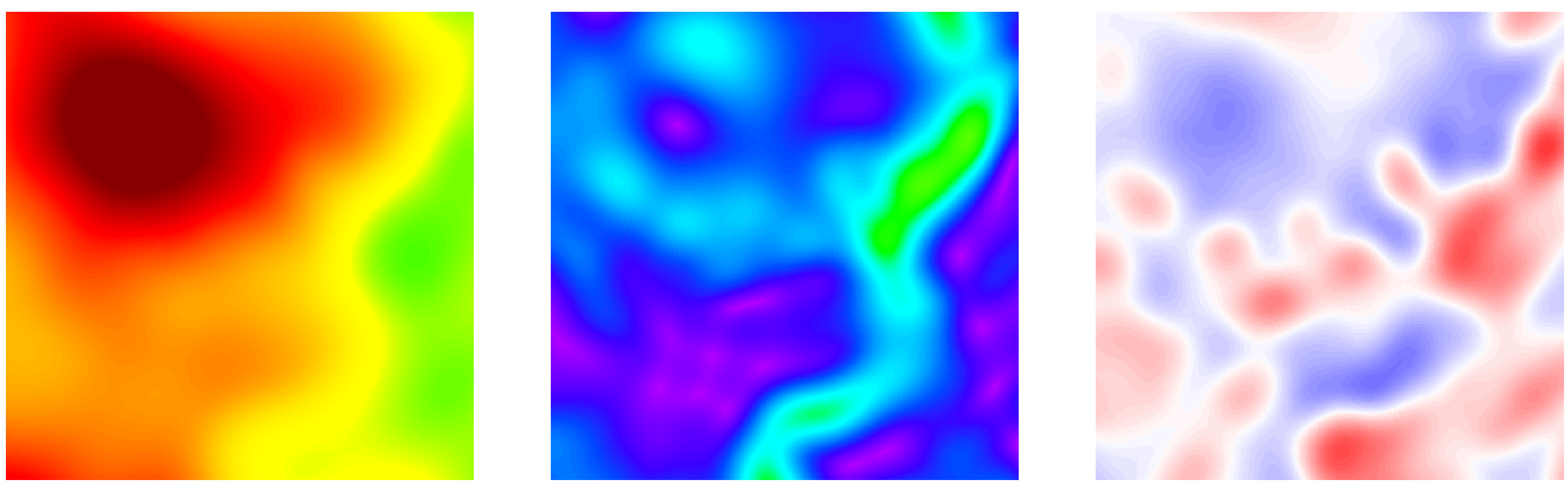


SSH, Geostrophic Speed and Geostrophic Relative Vorticity Smoothed with Filter Cutoff of 100 km

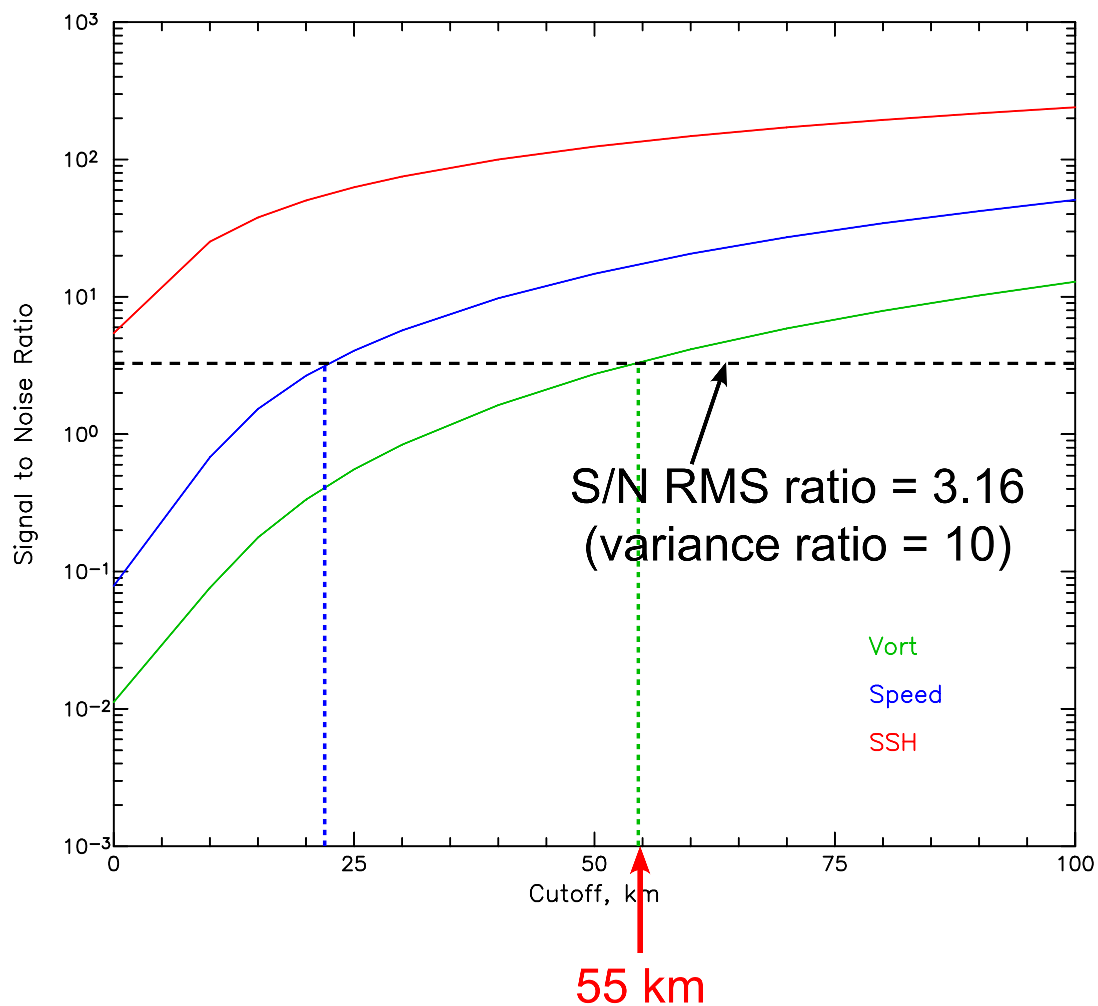
Error Free Measurements



With 2.74 cm RMS Measurement Errors



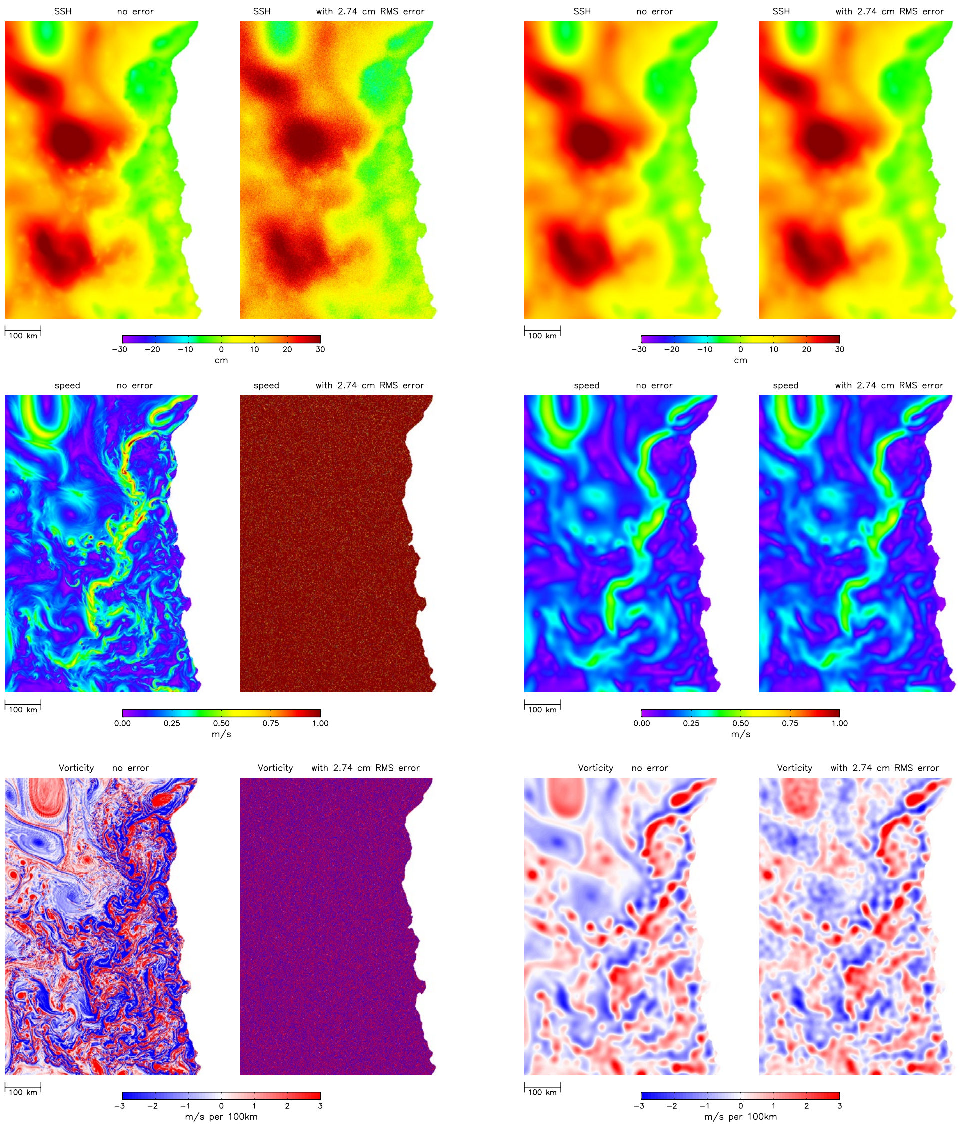
Signal-to-Noise RMS Ratios for SWOT Estimates of SSH, Geostrophic Speed and Geostrophic Relative Vorticity



SSH, Geostrophic Speed and Geostrophic Relative Vorticity with and without 2.74 cm Noise, Unsmoothed and Smoothed

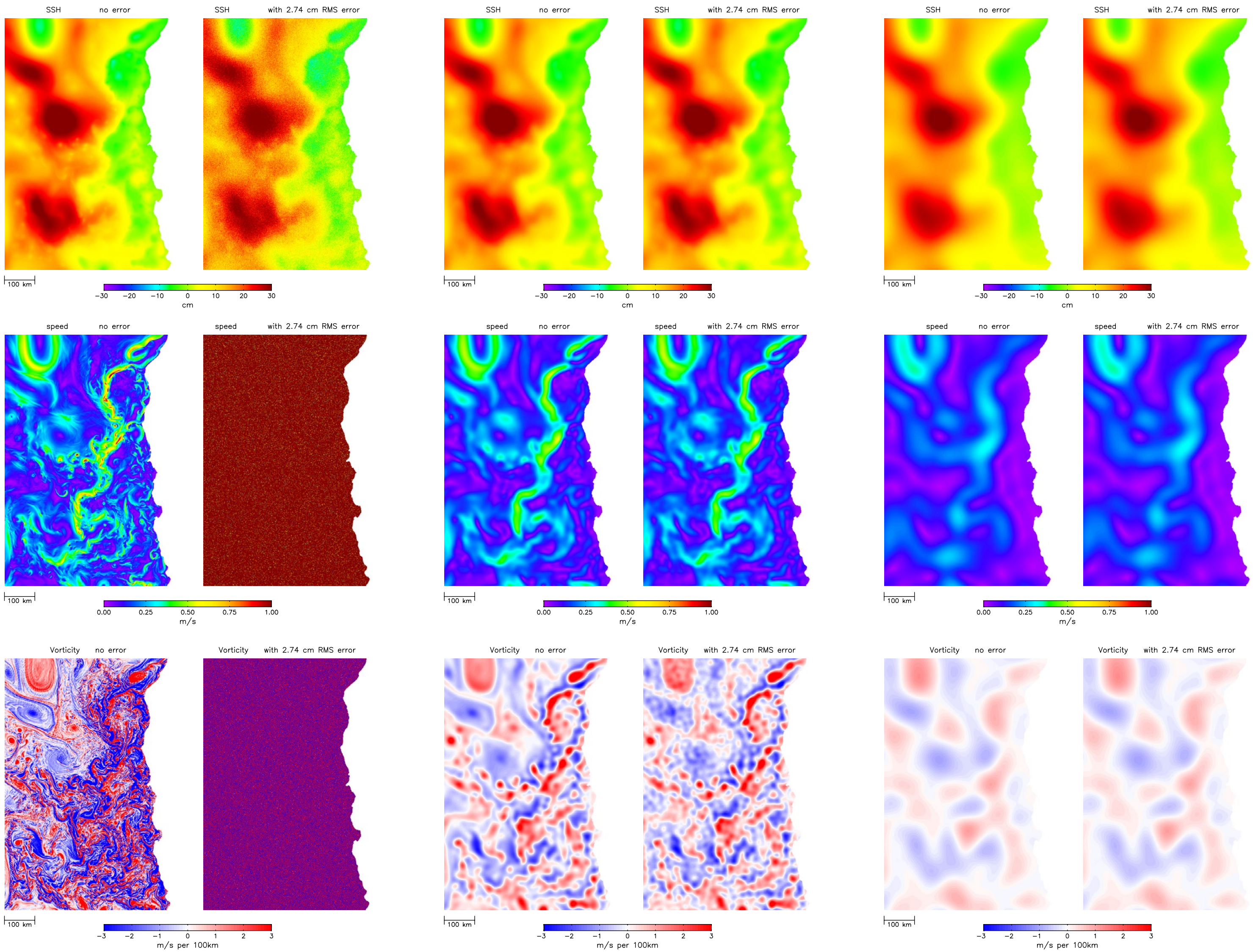
Unsmoothed

Filter Cutoff of 60 km



SSH, Geostrophic Speed and Geostrophic Relative Vorticity with and without 2.74 cm Noise, Unsmoothed and Smoothed

Unsmoothed | Filter Cutoff of 60 km | Filter Cutoff of 200 km



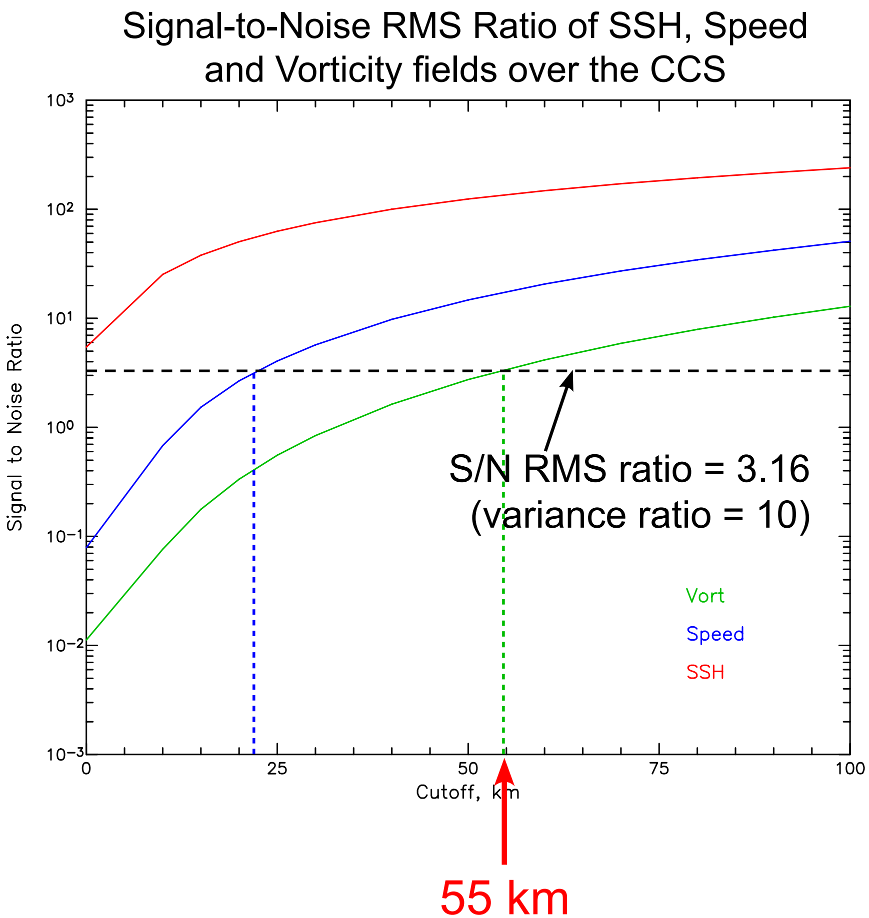
Conclusions

- The RMS of the uncorrelated measurement errors of the onboard processed SWOT estimates of SSH is 2.74 cm.
- Energetic submesoscale variability looks very pretty dull in SSH compared with vorticity.
- The double differentiation of SSH to get geostrophic relative vorticity amplifies the small-scale uncorrelated measurement errors.
- Depending on the choice of S/N ratio, the resolution capability for SWOT estimates of geostrophic relative vorticity is somewhere between 30 and 60 km wavelength.
  - While perhaps disappointing, even a coarse resolution of 60 km is a major improvement over the present resolution capability of about 200 km wavelength.

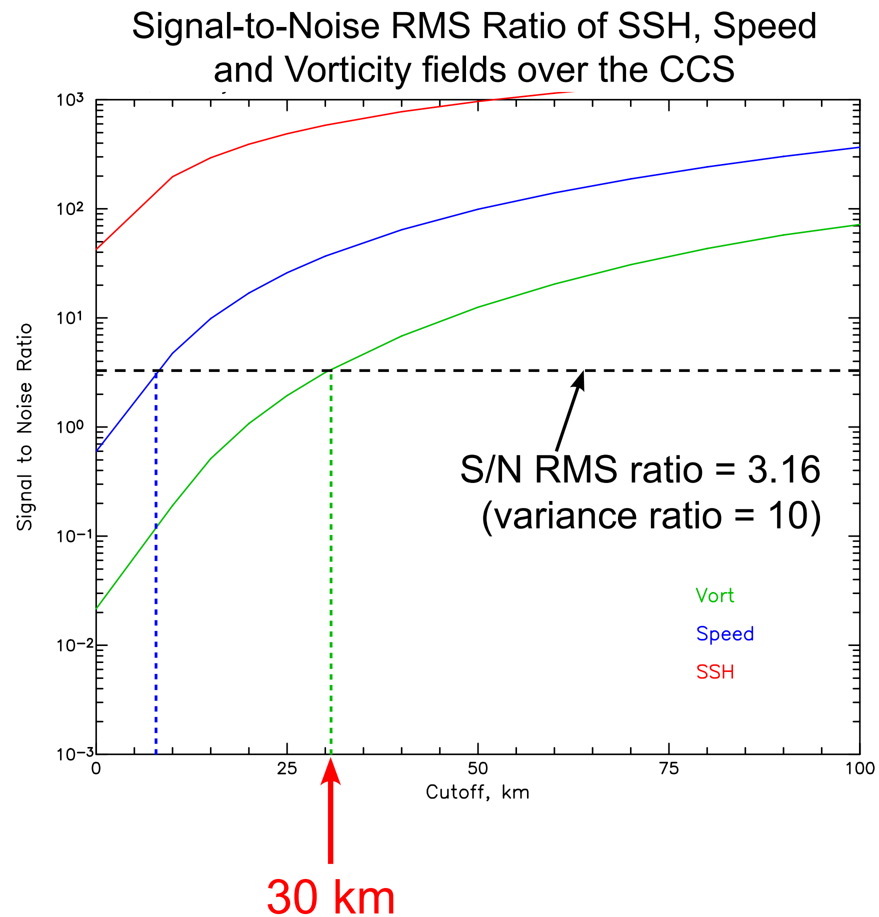
Extra Slides

Signal-to-Noise RMS Ratios for SWOT Estimates of SSH, Geostrophic Speed and Geostrophic Relative Vorticity

Spatial Smoothing Only



Spatial Smoothing and 30-Day Averaging



A Complete Analysis of Ekman Pumping (Stern, 1965)

In the conventional view, the vertical velocity from wind-driven Ekman pumping is

$$w_{Ek} \;=\; \frac{1}{\rho_0}\nabla\times\left(\frac{\vec{\tau}}{f}\right),$$

where ρ_0 is the water density, f is the planetary vorticity and $\vec{\tau}$ is the wind stress.

Stern (1965, *Deep-Sea Research*) shows that the planetary vorticity f should be replaced with the *absolute vorticity* $(f+\zeta)$. The "Stern-Ekman pumping" velocity is

$$w_{SE} \;=\; \frac{1}{\rho_0}\nabla\times\left(\frac{\vec{\tau}}{f+\zeta}\right) \;\approx\; \frac{\nabla\times\vec{\tau}}{\rho_0 f} \;+\; w_\beta \;+\; w_\zeta$$

where ρ_0 is the water density, f is the Coriolis parameter, ζ is the relative vorticity of surface currents, $\vec{\tau}$ is the surface stress with components τ_x and τ_y.

The Ekman pumping components w_β and w_ζ arise from gradients of f and ζ and are defined by

$$w_\beta \;\equiv\; \frac{\beta\,\tau_x}{\rho_0 f^2} \qquad\qquad \boxed{w_\zeta \;\equiv\; \frac{1}{\rho_0 f^2}\left(\tau_x\frac{\partial\zeta}{\partial y} \;-\; \tau_y\frac{\partial\zeta}{\partial x}\right)}$$

For the submesoscale and mesoscale variability of interest here,

$$\beta = \frac{df}{dy} \;\ll\; \frac{\partial\zeta}{\partial x},\;\frac{\partial\zeta}{\partial y} \quad\Rightarrow\quad w_\beta \ll w_\zeta$$

We will therefore neglect w_β.

Assumptions of the Stern (1965) Theory

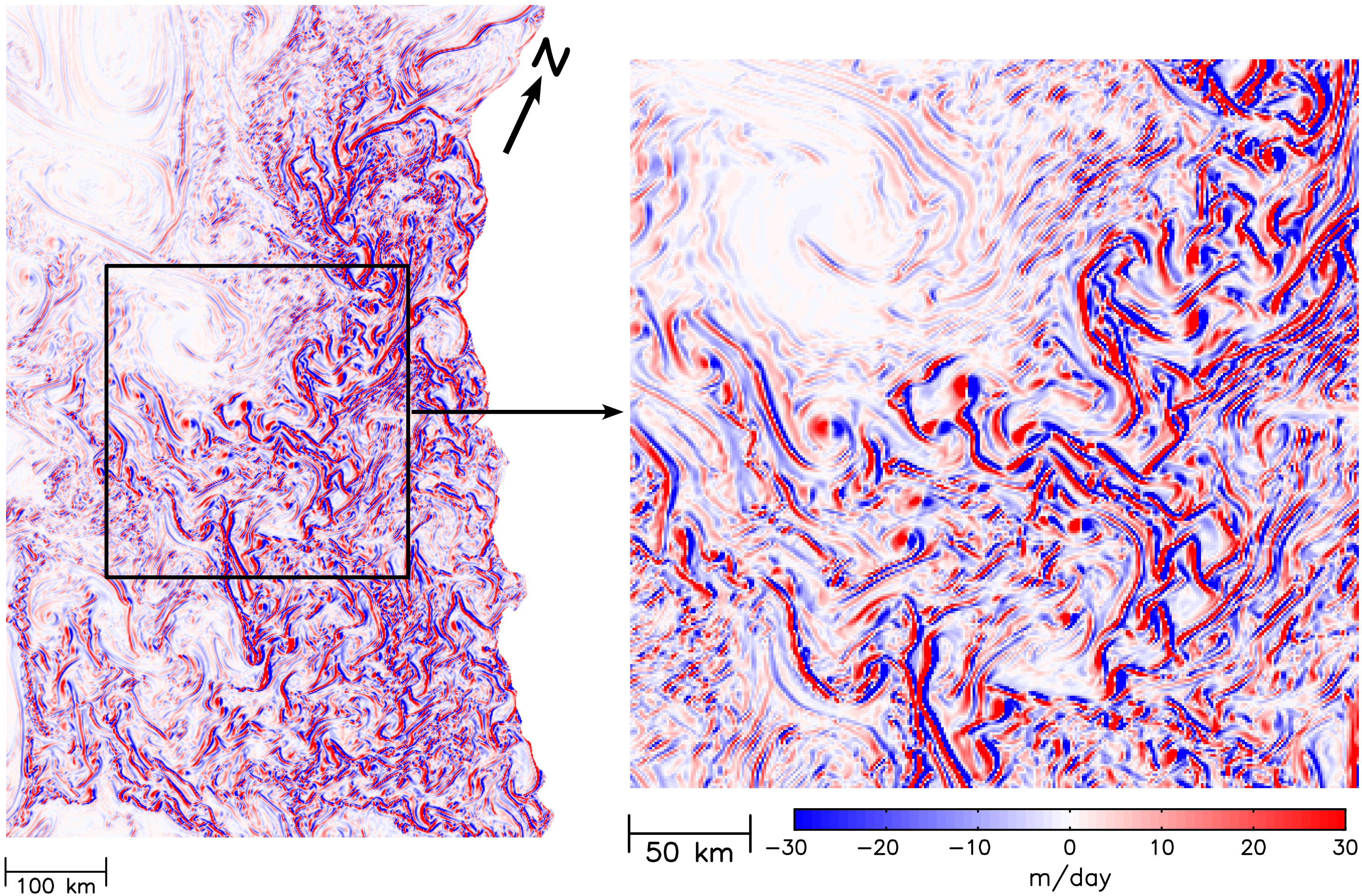
$$w_{SE} = \frac{1}{\rho_0} \nabla \times \left(\frac{\vec{\tau}}{f + \zeta} \right) \approx \frac{\nabla \times \vec{\tau}}{\rho_0 f} + \underbrace{\frac{1}{\rho_0 f^2} \left(\tau_x \frac{\partial \zeta}{\partial y} - \tau_y \frac{\partial \zeta}{\partial x} \right)}_{w_\zeta}$$

- The geostrophic Rossby number $\epsilon_g = U_g / fL$ is small, where L is a characteristic horizontal scale and U_g is a characteristic speed of surface geostrophic velocity. Small geostrophic Rossby number is equivalent to assuming that $\zeta << f$.

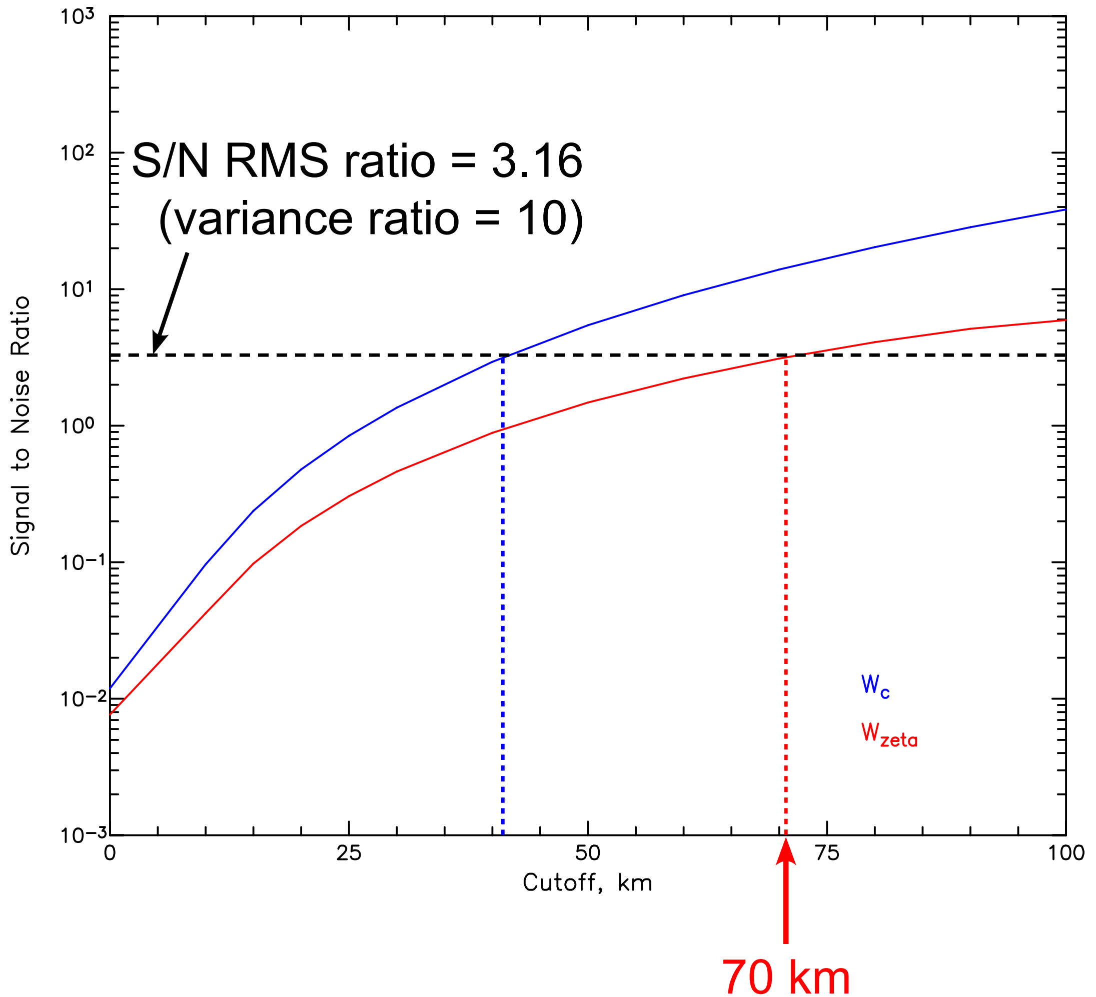
- The Ekman Rossby number $\epsilon_E = U_E / fL$ is also small, where L is again the horizontal scale of the submesoscale variability and U_E is a characteristic speed of surface Ekman currents.

- The geostrophic vorticity ζ is vertically uniform within the boundary layer. This assumption may be violated as the horizontal length scale L becomes small, and it may therefore be necessary to account for vertical shear of the geostrophic surface currents.

Vorticity Gradient Induced Ekman Pumping w_ζ from a Model of the CCS with 0.5 km Grid, Smoothed to 1 km Resolution



Signal-to-Noise RMS Ratios for SWOT Estimates of Ekman Pumping Components w_c and w_ζ



Geostrophic Vorticity Gradient Induced Ekman Pumping w_ζ with and without SWOT 2.74 cm Noise

Unsmoothed

Filter Cutoff of 70 km

Filter Cutoff of 200 km