

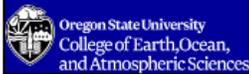
Eulerian Statistics for Fluid Trapping and Transport by SSH-Tracked Eddies: Preliminary Results

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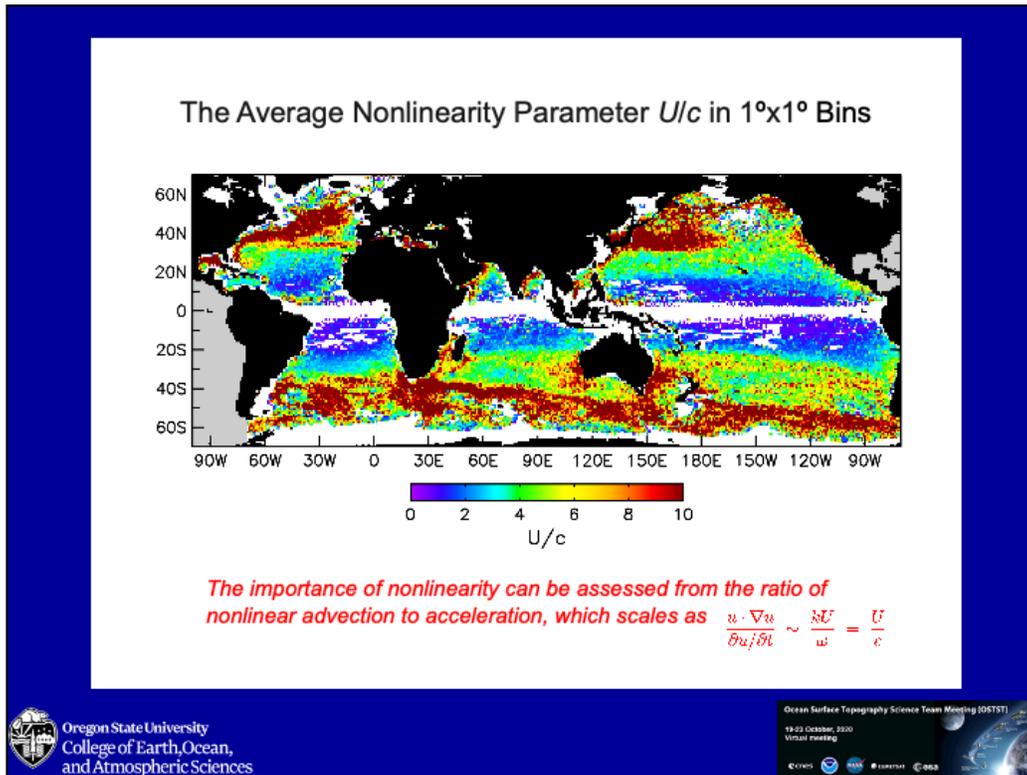
Ocean Surface Topography Science Team Virtual Meeting
October 2020



We present a progress report on the development of Eulerian statistics for efficient estimation of fluid trapping and transport by eddies identified and tracked from the global, gridded, multi-decadal, multi-altimeter SSH dataset.

Existing methods for computing Lagrangian transport of fluid by mesoscale eddies primarily depend on explicit computation of Lagrangian trajectories or a large amount of related Lagrangian information, and are computationally intensive. We seek an efficient alternative method for estimating this transport that is based on Eulerian statistics and can be applied conveniently to large datasets such as the global, gridded, multi-decadal, multi-altimeter SSH products available from CLS/DUACS/AVISO.

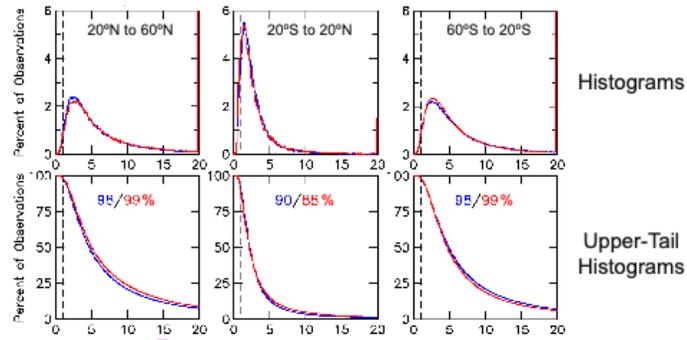
The method makes use of Eulerian estimates of the nonlinearity of SSH-tracked eddies. A progress report on this work is provided here. An anticipated future refinement is the incorporation of other Eulerian statistics that quantify the temporal variability or coherence of the tracked eddy structures, which can have a strong influence on fluid trapping and Lagrangian transport.



A traditional nonlinearity parameter for nonlinear waves, denoted U/c , can be computed for each of the altimeter-SSH-tracked eddies. The parameter is the ratio of U , the speed of rotational, geostrophic flow around the eddy, to c , the eddy propagation speed. The U/c nonlinearity parameter is a traditional measure of the relative importance of fluid advection and wave propagation for general nonlinear waves, with advection becoming progressively more important as U increases for $U/c > 1$, and so can be expected to be relevant to the problem of determining the fluid trapping and transport by SSH-tracked eddies.

Distributions of Nonlinearity Parameter U/c in 3 Latitude Bands for Cyclonic and Anticyclonic Eddies

The characteristic fluid velocity U within the eddy interior is based on the average geostrophic speed around the SSH contour with maximum average geostrophic speed, and the translation speed c is computed from the eddy tracking.



For the extratropical eddies:

- 98% had $U/c > 1$
- 48% had $U/c > 5$
- 21% had $U/c > 10$

The nonlinearity parameter U/c computed for the SSH-tracked eddies has a pronounced latitude dependence, with larger values at higher latitudes.

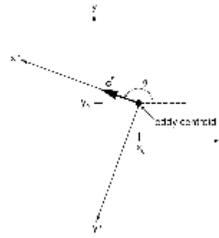
Region of Trapped Fluid Within an Eddy

Consider an eddy at time $t = t_0$ centered at location $(x, y) = (x_0, y_0)$ in an Eulerian reference frame. Define a rotated and translating reference frame with origin at the eddy center, x' axis aligned in the direction of the instantaneous propagation vector \mathbf{c} , and y' axis oriented 90° counterclockwise from the x' axis so as to form a right-handed coordinate system. The two coordinate systems at time $t - t_0$ are related by

$$x' = (x - x_0) \cos \theta + (y - y_0) \sin \theta - (t - t_0)c$$

$$y' = -(x - x_0) \sin \theta + (y - y_0) \cos \theta,$$

where $\theta = \tan^{-1}(c_y/c_x)$ for components c_x and c_y of the eddy propagation vector \mathbf{c} .



In terms of sea-surface height h , streamlines in a Lagrangian reference frame moving with the eddy are

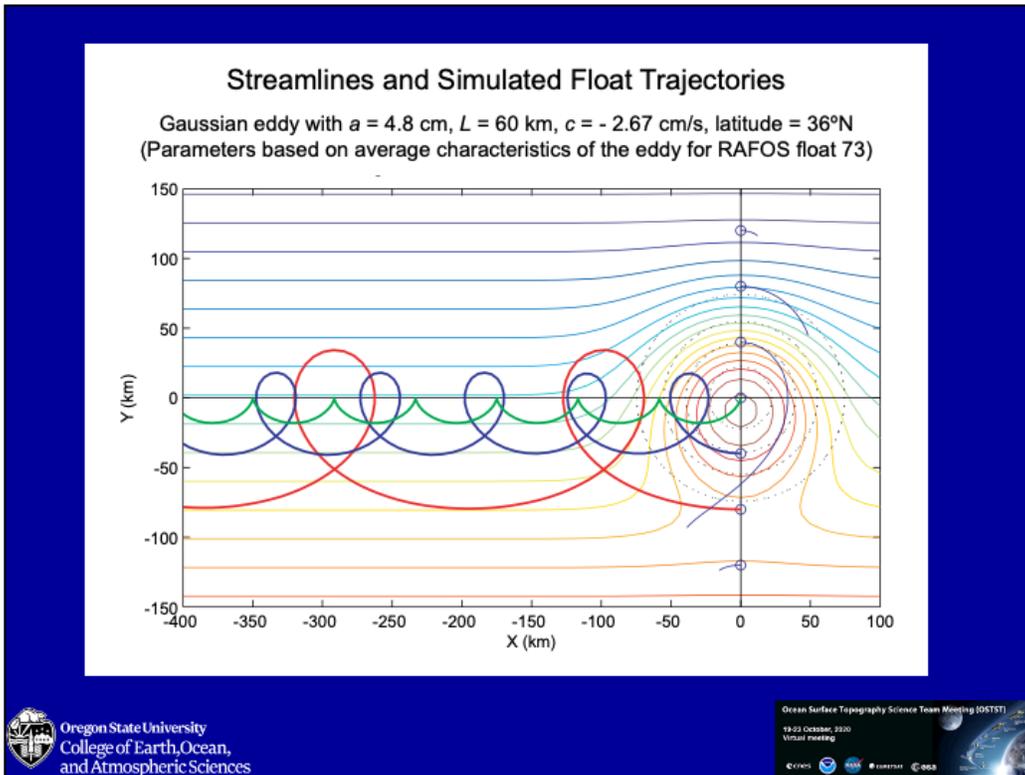
$$\psi'(x', y', z, t_0) = \frac{g}{f} \left[h(x', y', t_0) + \frac{cf}{g} y' \right],$$

In the translating reference frame, *a propagating eddy thus effectively imposes a linearly varying SSH parallel to the y' axis with a slope of cf/g .*

Compared with closed contours of SSH, closed streamlines in the moving reference frame are displaced:

- equatorward for anticyclonic eddies (positive SSH)
- poleward for cyclonic eddies (negative SSH).

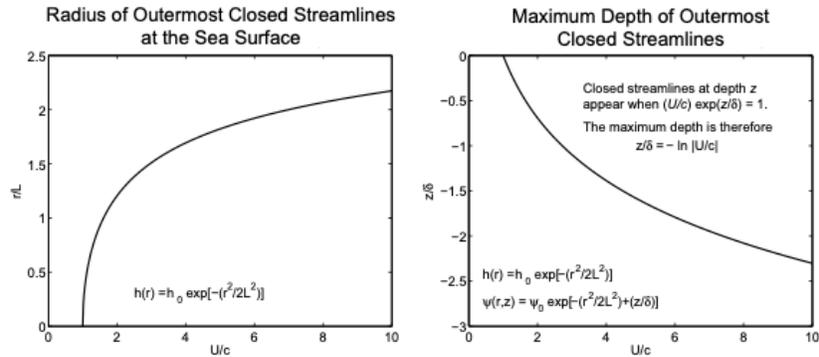
For the case of a perfectly coherent, steadily translating eddy on a resting background state, the region of trapped fluid can be computed exactly by transforming the stream function to a frame of reference moving with the eddy and identifying the region of closed streamlines in the co-moving frame. For a fixed eddy shape, the size of the trapping region is determined by the value of the nonlinearity parameter U/c .



The characteristic structures of fluid trajectories inside and outside of the trapping region of a propagating eddy are shown, for the case of a perfectly coherent, steadily translating eddy on a resting background state.

The Effects of Nonlinearity on the Region of Trapped Fluid for Gaussian Eddies that Decay Exponentially with Depth

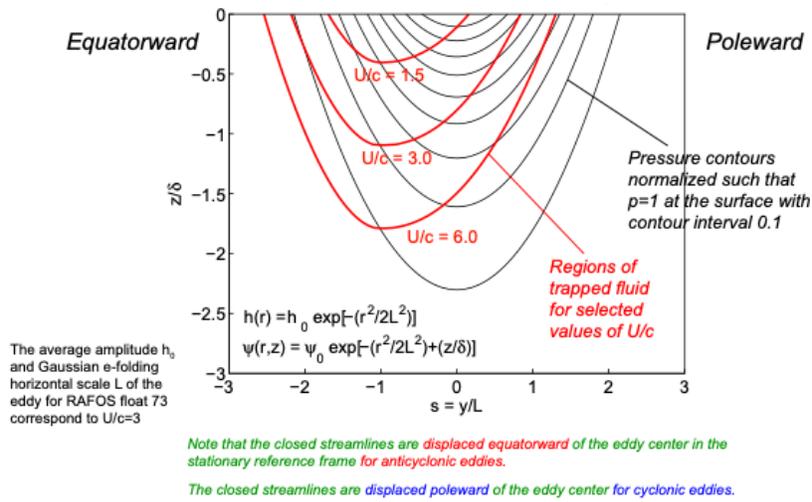
The importance of nonlinearity can be assessed from the ratio of nonlinear advection to acceleration, which scales as $\frac{u \cdot \nabla u}{\partial u_i / \partial t} \sim \frac{kL^2}{\omega} = \frac{U}{\epsilon}$



When normalized by the horizontal scale L and the e -folding depth scale δ of the eddy, the maximum radius and maximum depth of closed streamlines both depend only on the nonlinearity parameter U/c .

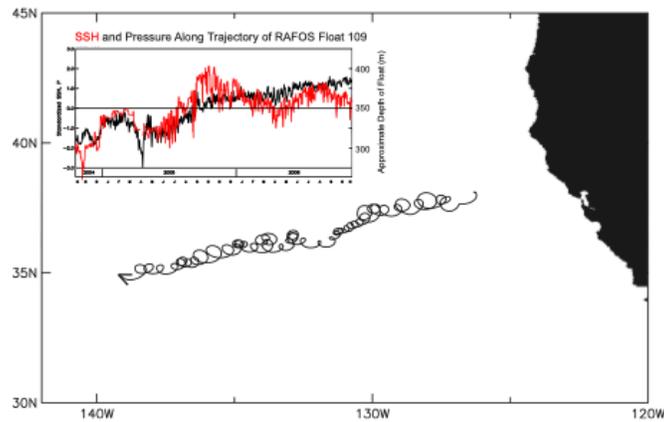
Some basic parameter dependencies of the region of trapped fluid are shown, for the case of a perfectly coherent, steadily translating eddy structure with exponential depth decay.

Vertical Structure of Closed Streamlines for a Gaussian Anticyclonic Eddy that Decays Exponentially with Depth



The trapping region will generally decrease with depth, for surface-intensified eddies, as in this analytical example with exponential depth decay.

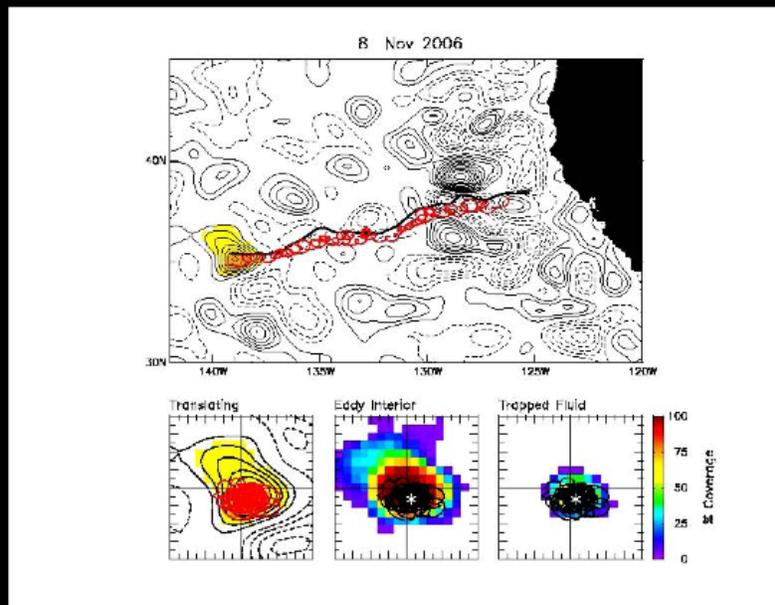
Trajectory of NPGS RAFOS Float 109 in the California Current
31 August 2005 - 8 November 2006
(Courtesy of Curt Collins, NPGS)



The depth range of the float was about 310-380 m.

An example of a RAFOS float (Curt Collins, Naval Postgraduate School) trapped in a mesoscale eddy in the California Current System.

Animation of NPGS RAFOS Float 109 in the California Current
31 August 2005 - 8 November 2006
(Courtesy of Curt Collins, NPGS)

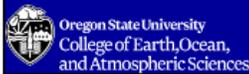


This is the final frame of an animation of the RAFOS trajectory and altimeter SSH that is not included in the presentation file but is available from the authors on request.

Future extensions

The preliminary results shown here all illustrate results for a perfectly coherent, steadily translating eddy in a resting background flow. It is well known, however, that time-dependence in the eddy and background flows will affect the coherent transport.

A natural extension of this work that we hope to pursue is to incorporate Eulerian measures of the temporal variability of the flow, to refine the estimates of trapped fluid and Lagrangian transport. These may be derived directly from altimeter SSH statistics or possibly also from comparison with dynamical models of mesoscale circulation.



A natural basis for the model comparison is the ocean mesoscale regime of the quasi-geostrophic model that was recently identified by Samelson et al. (2019), as summarized briefly in the “Salient Results from OSTST 2017-2020” contribution by the same authors.

Samelson, R. M., D. B. Chelton, and M. G. Schlax, 2019. The ocean mesoscale regime of the reduced-gravity quasi-geostrophic model. *J. Phys. Oceanogr.*, 49, 2469–2498, DOI: 10.1175/JPO-D-18-0260.1; see also links to informal errata and source code at

http://www-poa.coas.oregonstate.edu/~rms/ms/jpo2019omrqq_jpo-d-18-0260.1_errata_eqs_26_27_Fig16.pdf

and

https://github.com/rsamelson/quasigeostrophic_spectral_layer_model/tree/master/qg_1layer_dp.