A Fast Convolution based Model for SAR Altimetry Waveforms and their Retracking

Christopher Buchhaupt (TU Darmstadt), Luciana Fenoglio-Marc (TU Darmstadt), Salvatore Dinardo (SERCO/ESRIN), Remko Scharroo (EUMETSAT), Matthias Becker (TU Darmstadt)

1 Introduction

In the last few years several algorithms have been developed to extract geophysical information of the sea surface from SAR altimetry data. These can be divided into full analytical (SAMOSA2 [1]), semi analytical (DDA4 [2]) and numerical (CPP). Whereas the full analytical algorithms are fast but need a Look Up Table (LUT) to get correct results the semi analytic and numerical retrackers are slower and do not need any LUT.

Aim of this study is to develop a numerical algorithm to retrack SAR waveforms. We consider the received power by a double convolution formula (see Eq. 1)

$$P(\tau,x) = FSSR(\tau,x) * PTR(\tau,x) * PDF(\tau) \tag{1}$$

whereas $P(\tau,x)$ is the received power at time τ and at along track position x, $FSSR(\tau,x)$ the flat sea surface response, $PTR(\tau,x)$ the point target response, which is a double $sinc^2$ in both directions and $PDF(\tau)$ the probability density function of the sea surface elevation which is a Gaussian.
By using the convolution theorem Eq. 1 can be simplified by a two dimensional Fourier transform

$$\hat{\hat{P}}(f,\xi) = F\hat{\hat{S}}SR(f,\xi) \cdot P\hat{\hat{T}}R(f,\xi) \cdot P\hat{D}F(f) \tag{2}$$

whereas f the frequency in the range direction and ξ a variable which is received after the Fourier transform in the along track direction.

The Fourier transforms of the PTR is a double triangular window in both directions and the PDF is a Gaussian. In the next section a close form solution for $F\hat{\hat{S}}SR$ is developed.

2 Calculating the two dimensional Fourier transform of the FSSR

An analytical representation for the FSSR can be derived by using Eq. 23 from [1] and assuming an infinite window width in the x direction, an infinite small pulse length and a SWH of zero meter.

For a better visualization $\Gamma(x,y) = G^2(x,y) \cdot \sigma_(x,y)$ is set. The convolution term in Eq. 3 is applied to undo the range cell migration correction.

$$\begin{aligned} FSSR(\tau,x) &= \frac{A}{G_0^2 \cdot \sigma_0} \int_{-\infty}^{\infty} \delta(\hat{z}) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Gamma(\hat{x},\hat{y}) \cdot \delta\left(k-k_l\right) \cdot \delta(\hat{x}-x) d\hat{x} d\hat{y} d\hat{z} * \delta\left(\tau - \frac{\alpha}{ch}x^2\right) \\ &= A \cdot U(\tau) \cdot e^{-\frac{\gamma_x}{h^2}(x-x_p)^2 - \frac{c\tau}{\alpha h}(\gamma_y+\nu) - \frac{\gamma_y}{h^2}y_p^2 - \frac{\nu}{h^2}x^2} \cdot \frac{\cosh\left(\frac{2y_p\gamma_y}{h^2}\sqrt{\frac{c\tau h}{\alpha}}\right)}{\sqrt{ch\tau/\alpha}} * \delta\left(\tau - \frac{\alpha}{ch}x^2\right) \end{aligned} \tag{3}$$

whereas $\gamma_x = \frac{8\ln(2)}{\Theta^2_{3dBx}}$ is the propagation coefficient in x direction, $\gamma_y = \frac{8\ln(2)}{\Theta^2_{3dBy}}$ the propagation coefficient in y direction, $x_p = h \cdot \Theta_p$ the mispointing in x direction, $y_p = -h \cdot \Theta_r$ the mispointing in y direction and ν the inverse of the sea surface mean square slope

In the next step the Fourier transform in the along track direction is calculated.

$$F\hat{S}SR(f,x) = K \cdot \sqrt{\frac{\alpha\pi}{ch}} \cdot \exp\left\{-\left(\frac{\gamma_x}{h^2} + \frac{\nu}{h^2} + \frac{2\pi i \alpha f}{ch}\right)x^2 + 2\frac{\gamma_x}{h^2}x_p x + \Theta_r^2 \gamma_y \frac{c}{\alpha h s}\right\} \tag{4}$$

here $K = A \cdot e^{-\gamma_x \Theta_p^2 - \gamma_y \Theta_r^2}$ denotes the Amplitude scaled by mispointing

The Fourier transform in the time domain can be handled as a Laplace transform by setting $s = \frac{c \cdot (\gamma_y+\nu)}{\alpha h} + 2\pi i f$ which leads to Eq. 5

$$F\hat{\hat{S}}SR(f,\xi) = K \cdot \pi \cdot \frac{\exp\left\{\frac{a^2}{b+s} + \frac{\beta}{s}\right\}}{\sqrt{s \cdot (b+s)}} \tag{5}$$

whereas a, b and β are scalar values which contain the mispointing, the sea surface mean square slope and the antenna parameters.

$$a = \gamma_x \Theta_p \sqrt{\frac{c}{\alpha h}} - \pi i \sqrt{\frac{ch}{\alpha}}\xi \tag{6}$$

$$b = \frac{c}{\alpha h}(\gamma_x - \gamma_y) \tag{7}$$

$$\beta = \gamma_y^2 \Theta_r^2 \frac{c}{\alpha h} \tag{8}$$

The result for the FSSR in the frequency/ξ domain in Eq. 5 is quite easy to handle and to calculate. No extra simplifications were used.

3 Computation of a 20Hz SAR waveform

With the FSSR in the f/ξ domain the final waveform is obtained by the following steps.

1. Compute $F\hat{\hat{S}}SR$ for all wished f and ξ values
2. Multiply the $F\hat{\hat{S}}SR$ with the $P\hat{\hat{T}}R$ and compute a fast Fourier transform in the ξ direction
3. Apply the range cell migration correction and the $P\hat{D}F$ then compute the inverse fast Fourier transform in the frequency direction
4. Apply the Delay Doppler Mask to the stack
5. Average all beams to compute the 20Hz waveform

4 Cross-Validation

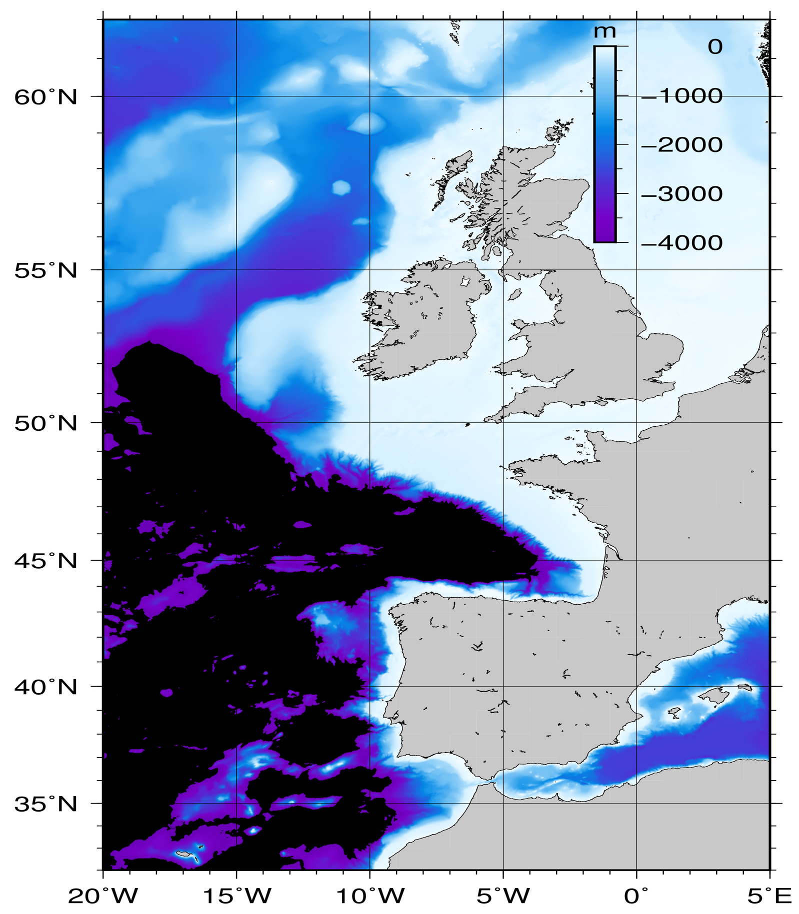
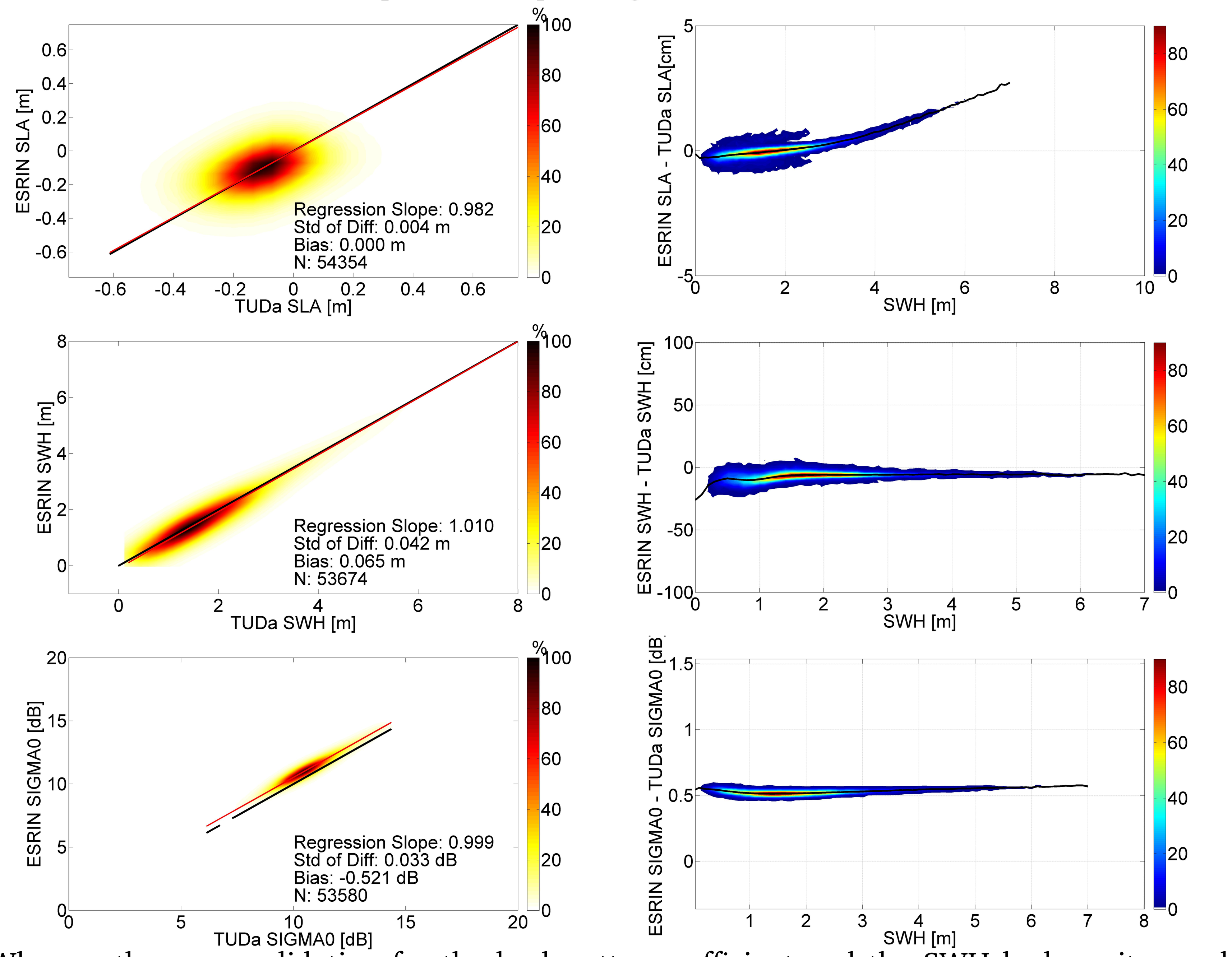


Figure 1: Chosen area for retracking

The model which have been developed in Section 2 is going to be tested in the Atlantic box. In this work only open ocean points (distance to coast > 10km) are selected to be retracked. The received parameters (Backscatter coefficient (σ_0), Sea level anomaly (SLA) and the wave height (SWH)) are averaged to 1Hz values by using a robust outlier detection with the median absolute deviation. For the 1Hz values following criteria are used:

1. Standard deviation (STD) of $\sigma_0 < 0.1dB$
2. STD of $SWH < 40cm$
3. STD of $SLA < 4cm$
4. Minimum number of points averaged 16

Our results TUDa are cross validated against the SAMOSA2 retracker by ESRIN. The figures on the left side shows scatter plots between the ESRIN and TUDa products and the right side the differences between both products depending on the SWH.



Whereas the cross validation for the backscatter coefficient and the SWH looks quite good (SIGMA0 just shows a bias of 0.5 dB) the SLA shows some dependency on SWH. It is assumed that this error is related to the treatment of thermal noise in the retracking algorithm. To overcome this problem further investigations are needed.

5 Conclusion

- A numerical SAR Retracker was developed.
- The results for SWH and SIGMA0 are looking promising.
- SLA differences between the TUDa and the ESRIN products shows a dependency on the SWH.
- SIGMA0 have a bias of -0.521 dB → factor is missing in the model

References

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Institute of Geodesy
Technische Universität Darmstadt, Germany
buchhaupt@psg.tu-darmstadt.de

Ocean Surface Topography Science Team Meeting,
Reston, Virginia, USA, October 20-23, 2015