The development of a novel tidal analysis package and its application to surface currents

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Introduction: Tidal Harmonic Analysis

Two broad methods are frequently used to analyze tides: Spectral methods use the energy contained in specific frequency bands to diagnose the amplitude of tidal constituents. Harmonic analysis uses a least-squares fit to estimate the amplitude and phase of tidal energy at known tidal frequencies (Munk and Hasselman, 1964; Zetler et al., 1965).

The t_tide package (Pawlowicz et al., 2002) is a frequently used harmonic analysis package that uses ordinary least squares (OLS) to estimate tidal constituents, with several specialized packages inspired by or built on it (Leffler and Jay, 2009; Foreman et al., 2009; Codiga, 2011; Matte et al., 2013).

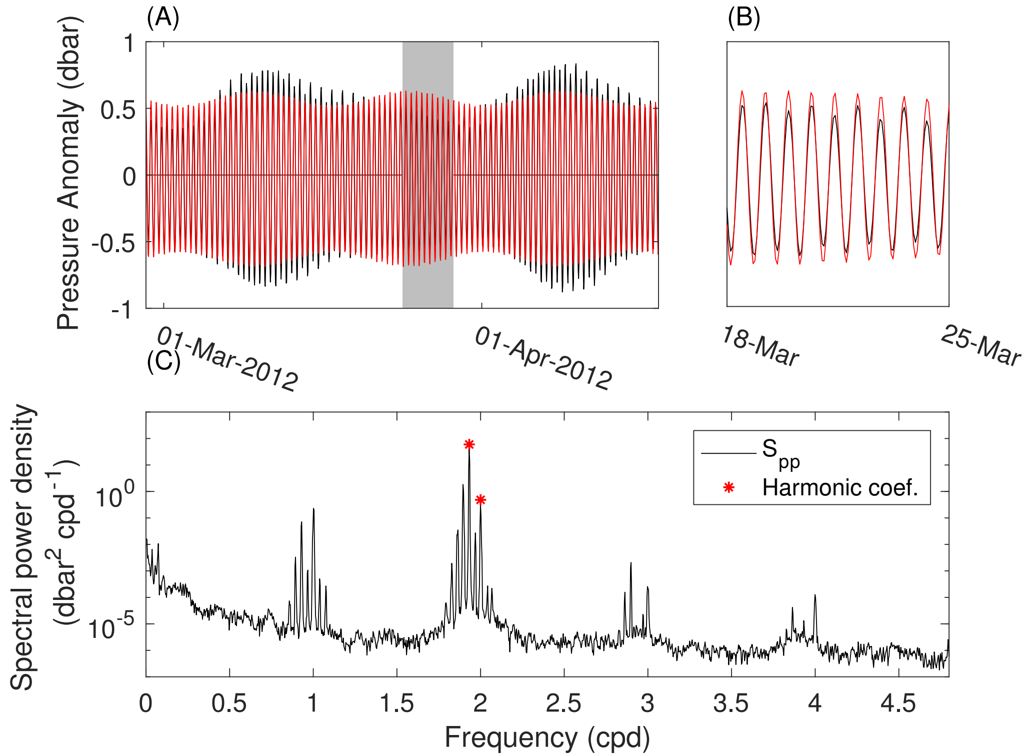
Ordinary Least Squares:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{r}$$

Where:
y = observed data
H = regressor matrix of basis functions (sinusoids at tidal frequencies)
x = model weights (unknown)
r = unmodeled residual

The solution that minimizes the variance of **r** is:

$$\mathbf{x} = \left(\mathbf{H}^{\mathbf{T}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathbf{T}}\mathbf{y}$$



Harmonic analysis of a bottom pressure time series (A, with B as the zoomed in gray segment), fit to two tidal frequencies. With substantial energy at many constituent frequencies (C), much variance remains unmodeled.

Non-stationary Tidal Energy and Structured Noise

Low-frequency processes can modulate tidal peaks such that energy is spread across a band of frequencies centered at the tidal frequency. Additionally, at frequencies outside the tidal bands, ocean data tend to be spectrally red, with greater power at lower frequencies (Munk et al., 1965). Together, these issues can affect harmonic analysis. OLS does not consider prior knowledge of time series statistics, which are often available.

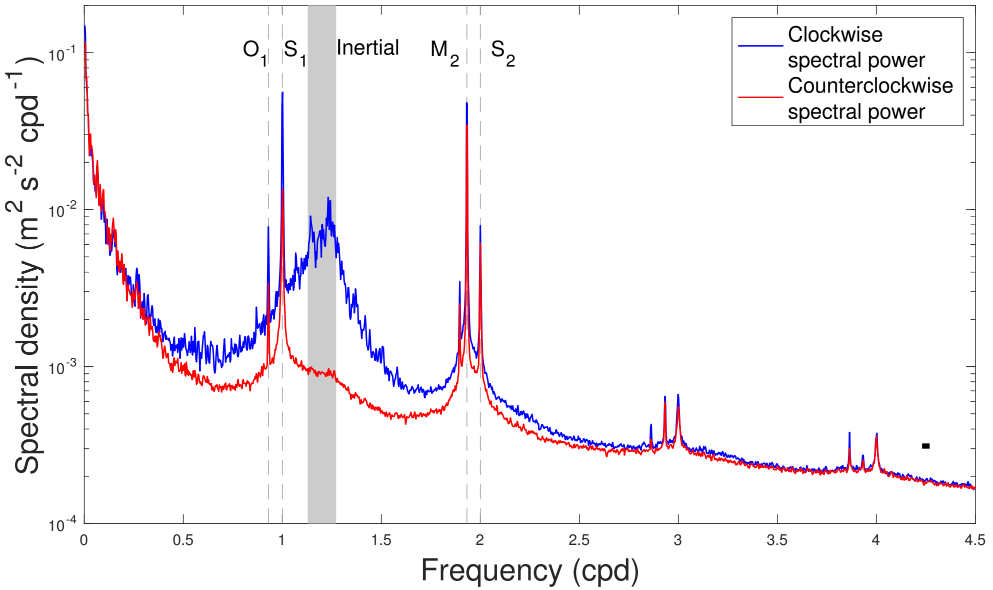
We have developed a new tidal analysis package (red_tide) that addresses the two-fold problem of non-stationary variability and structured (spectrally red) noise.

Bayesian maximum a posteriori estimate assuming linear and Gaussian statistics

The solution that minimizes the variance of **r** is:

$$\mathbf{x} = \left(\mathbf{H}^{\mathbf{T}}\mathbf{R}^{-1}\mathbf{H} + \mathbf{P}^{-1}\right)^{-1}\mathbf{H}^{\mathbf{T}}\mathbf{R}^{-1}\mathbf{y}$$

Where:
R = <**rr**$^{\mathbf{T}}$>, prior (assumed) covariance of **r**
P = <**xx**$^{\mathbf{T}}$>, prior covariance of **x**



Rotary spectrum of surface currents observed by high-frequency radar. This is an example of a tidally-driven record with substantial non-stationary energy (broad cusps) and an energetic red background.

Key Improvements of red_tide over Fourier Analysis

- Forced periodicity of model may be mitigated
- Frequency band limitation (band-limiting) eliminated
- Partition of variance between signal and noise may be prescribed, as well as the structured of both signal and noise
- Fourier limitations of evenly-spaced times and frequencies are lifted (this also applies to OLS)

Illustrative Cases: Step Function

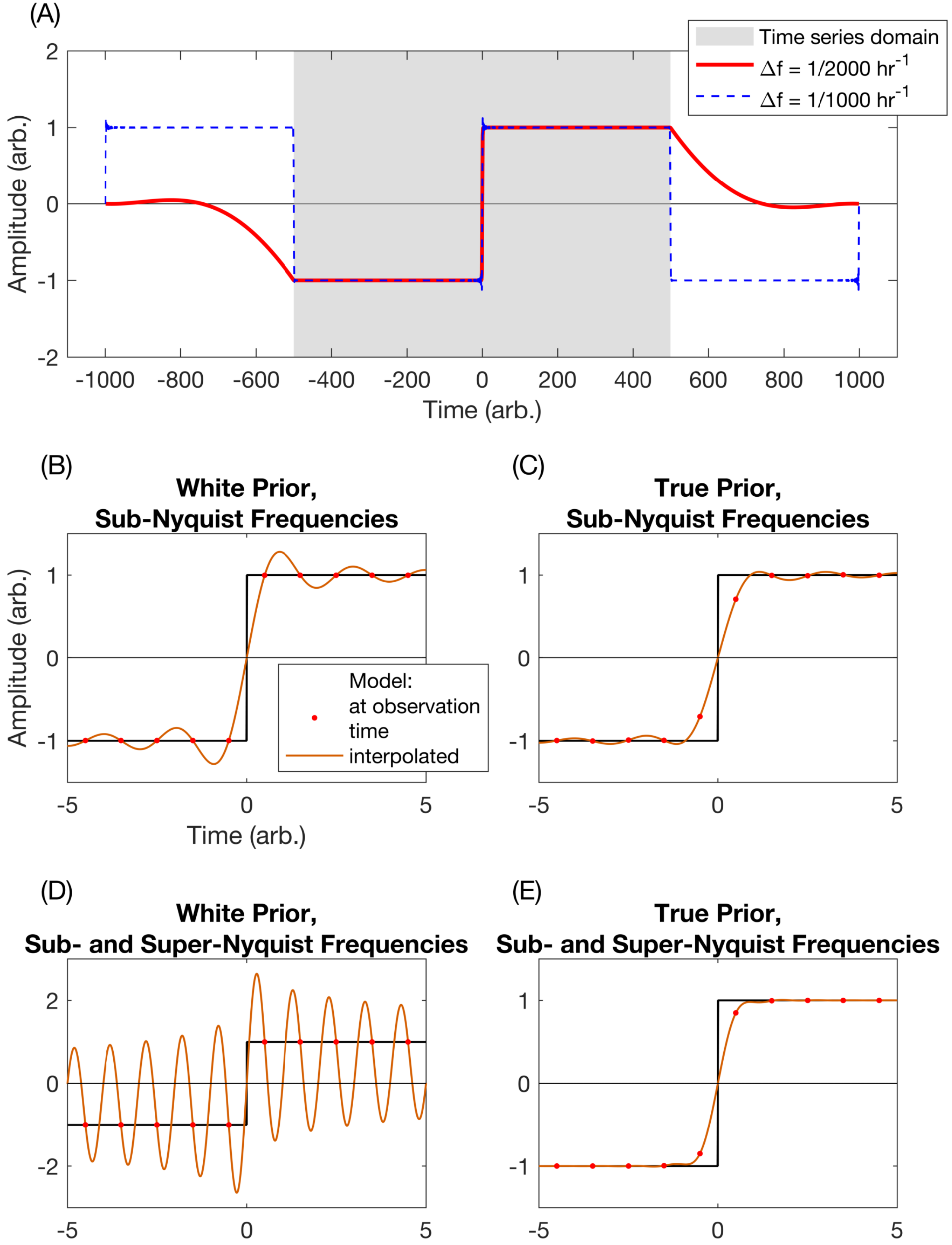
To demonstrate the effects of including prior statistics in harmonic analysis, we first analyze a step function. Near jump discontinuities, the Gibbs phenomenon is often observed when modeled by finite periodic bases. (A) The mitigation of model periodicity (and thus large, artificial jumps) by introducing additional frequencies. (B-E) Expanded view of jump discontinuity under different analysis regimes:

(B) model size <**xx**^{T}> assumed constant and no frequencies above the Nyquist frequency are modeled.

(C) model size assumed to be proportional to frequency^{-2} (the correct structure according to Fourier analysis) and no frequencies above the Nyquist frequency are modeled.

(D) model size <**xx**^{T}> assumed constant and frequencies above the Nyquist frequency are modeled. Without correct prior, model ambiguity results in large misfit.

(E) correct model size and frequencies above the Nyquist frequency are modeled. Gibbs phenomenon is minimized by fitting to "unresolvable" frequencies but allocating variance correctly using the prior.

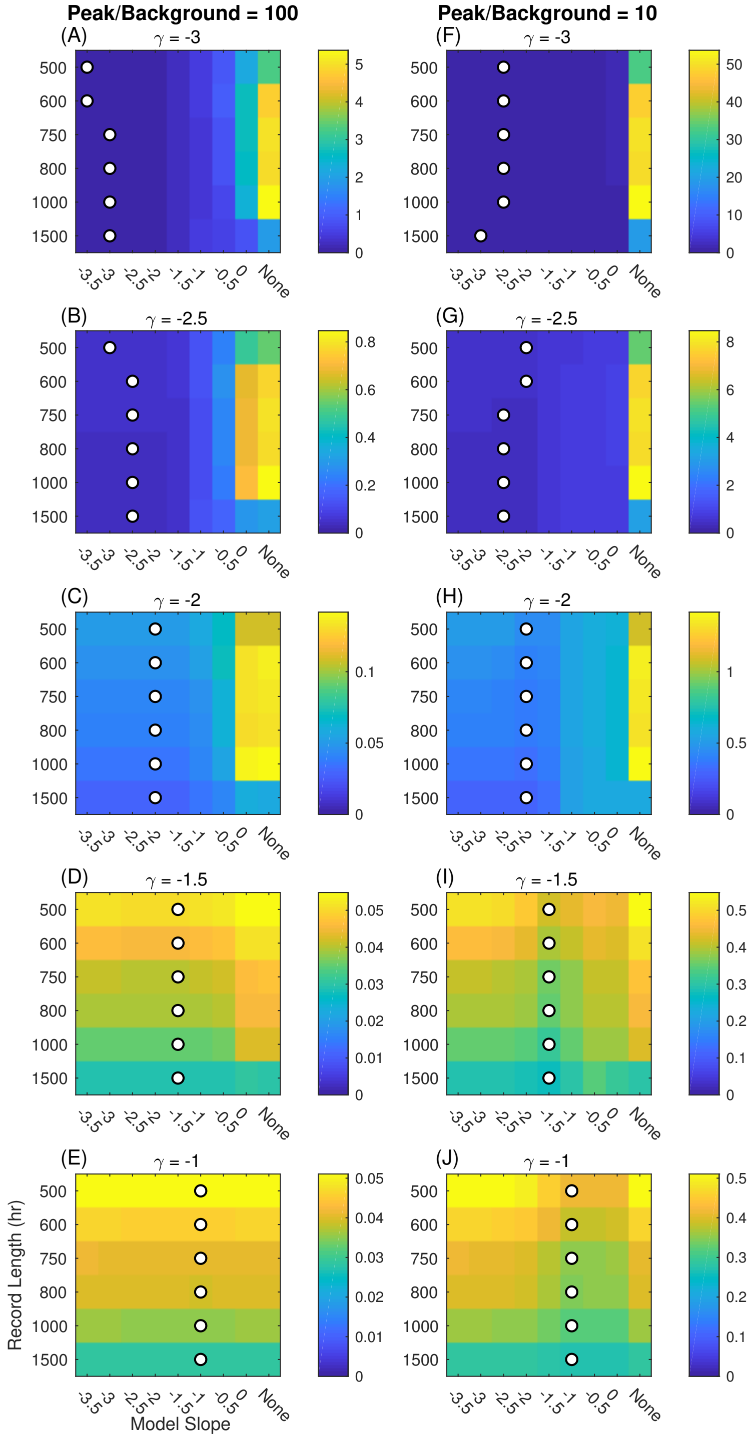


Illustrative Cases Continued: Varied Background Noise

To demonstrate the effects of background noise structured, we run Monte Carlo simulations while varying the structure of the true background noise and the background noise structure assumed by the model. The quantity plotted at the right is the normalized standard deviation of M_2 model coefficient difference from truth across N = 10000 simulations. White circles indicate the minimum (optimal) column for each row.

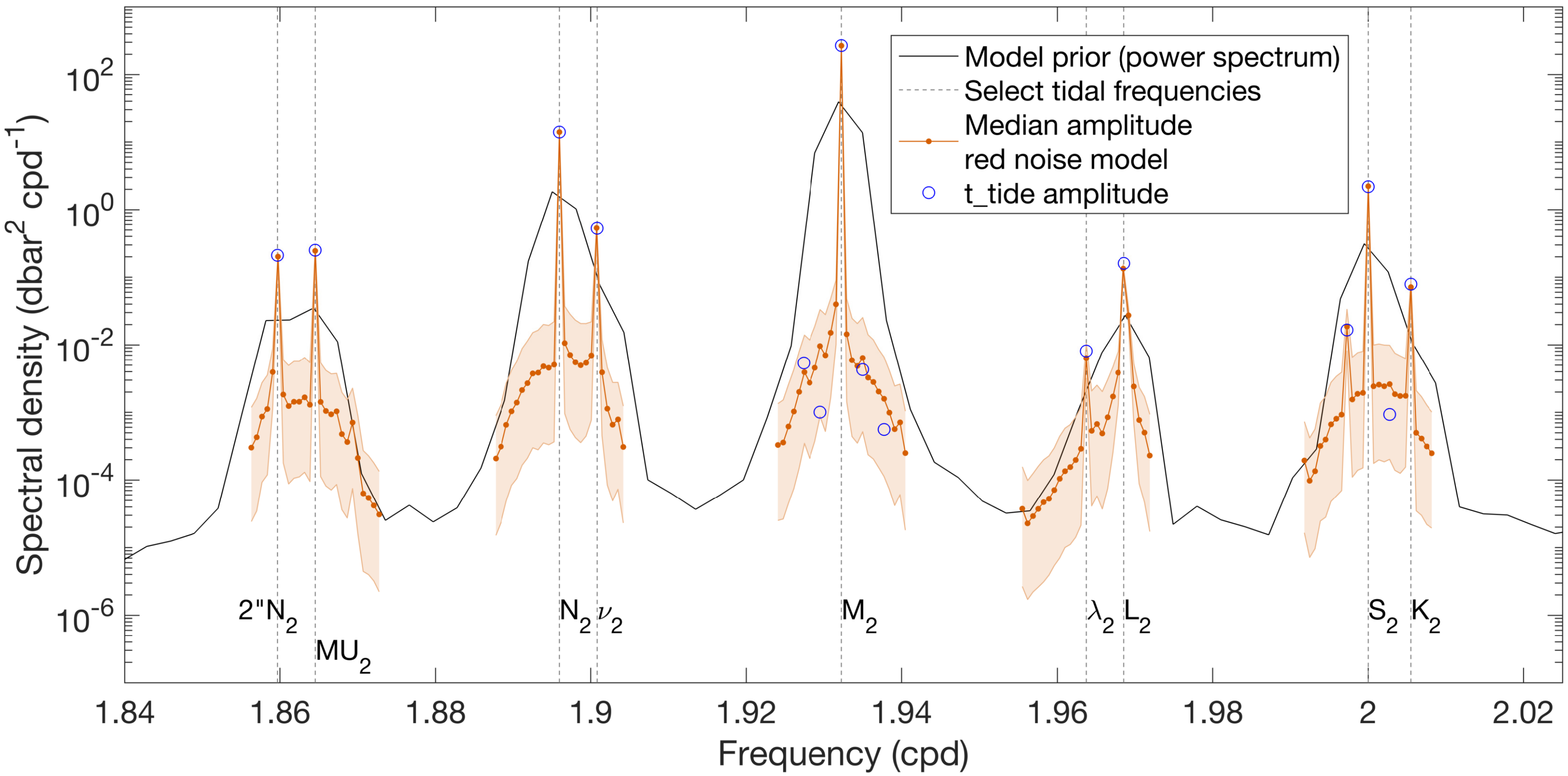
Each panel corresponds to a different "spectral background slope", the slope of the noise component in log-log spectral space. E.g. $\gamma = -1.5$ corresponds to **r** having a spectrum proportional to frequency$^{-1.5}$. The x-axis of each panel corresponds to the prior noise slope assumed by red_tide ("none" is the OLS approach with no assumptions about **r**). The y-axis corresponds to record length.

The inclusion of a prior for misfit **r** results in better estimates for **x** as this prior approaches the true value.



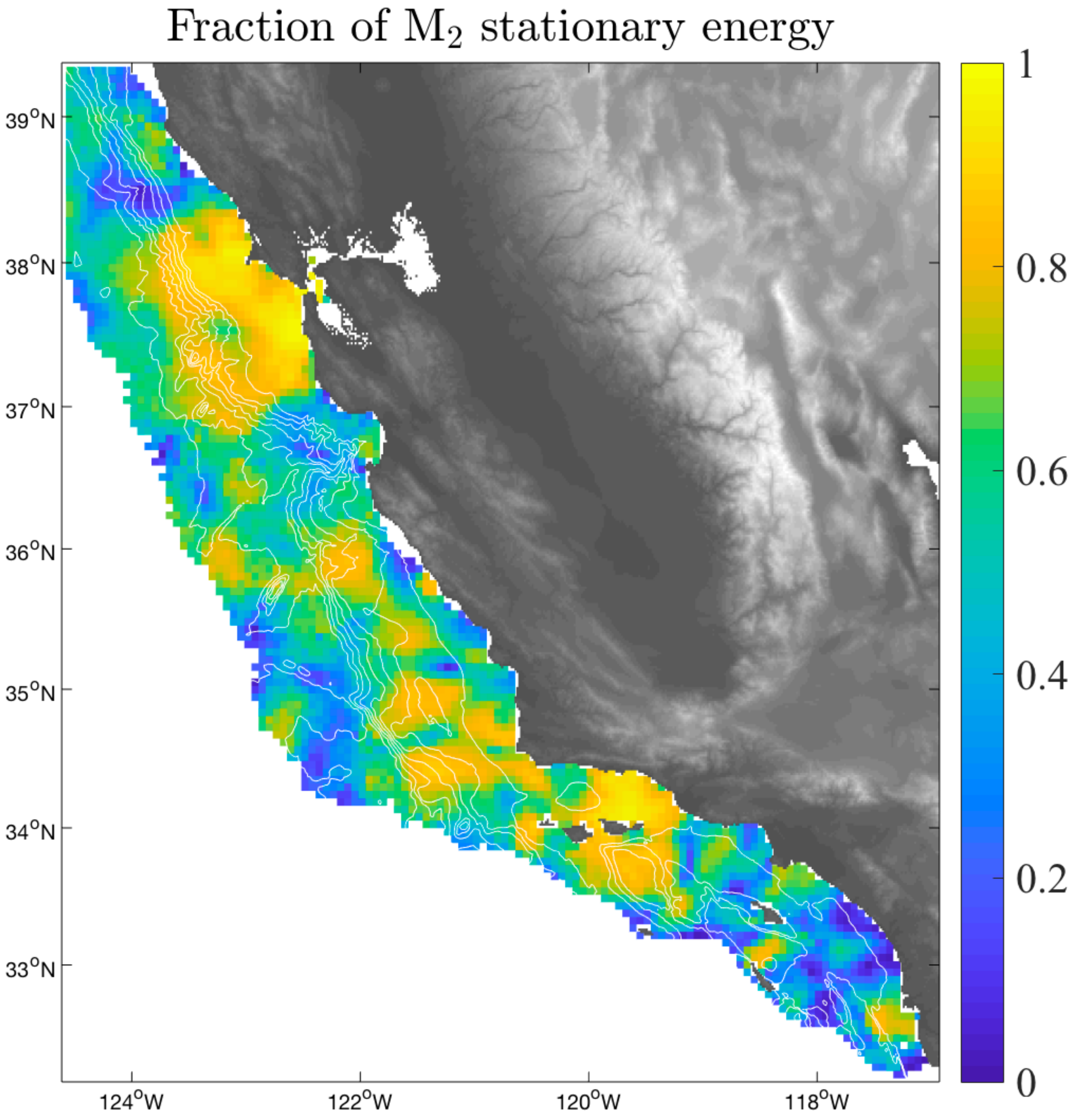
Application to Bottom Pressure Time Series

The output of red_tide matches that of t_tide when applied to highly stationary tidal records, like the multi-year bottom pressure record below. This is a zoom-in of the semi-diurnal band (similar results hold at other energetic bands). Notably, at low-energy frequencies red_tide can capture tidal cusp structure. Shading indicates 90% confidence intervals on red_tide coefficients.



Work in Progress: High-Frequency Radar Surface Current Analysis

Red_tide has been applied to mapped surface current records with highly non-stationary tidal components (Kachelein et al. OSM 2020). Notably, we can estimate the fraction of energy in an arbitrary frequency band that can be considered “non-stationary”. At right is a map from HFR of the fraction of energy in a band centered at M_2 that is not coherent with the pure M_2 constituent.



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